PRINCIPLES OF TOPOLOGICAL PSYCHOLOGY

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PREFACE

DR. WOLFGANG KÖHLER
Swarthmore College
Swarthmore, Pa.

DEAR KÖHLER:

This book is the result of a very slow growth.

I remember the moment when—more than ten years ago—it occurred to me that the figures on the blackboard which were to illustrate some problems for a group in psychology might after all be not merely illustrations but representations of real concepts. Much interested in the theory of science, I had already in 1912 as a student defended the thesis (against a then fully accepted philosophical dictum) that psychology, dealing with manifolds of coexisting facts, would be finally forced to use not only the concept of time but that of space too. Knowing something of the general theory of point sets, I felt vaguely that the young mathematical discipline "topology" might be of some help in making psychology a real science. I began studying topology and making use of its concepts, which soon appeared to me particularly fitted to the specific problems of psychology.

However, this undertaking expanded rapidly, forcing me to consider wider and wider fields of psychology and to face more and more involved problems. That is the reason why this book has seen quite a number of unfinished and unpublished editions, and why it does not yet contain the "vector psychology." The main difficulty has not been the mastering of the mathematical problems as such, at least insofar as the topological problems are concerned. After several attempts to employ the more complicated concepts of topology, I found it both sufficient and more fruitful to refer to the most simple topological concepts only. Vector psychology will, of course, require a more elaborate mathematical setup and will—in all probability—even make it necessary to enter a somewhat undeveloped field of mathematics. But the main difficulty was the dealing with problems which lie, so to say, *between* psychology and mathematics.

We know, since the theory of relativity at least, that empirical sciences are to some degree free in defining dynamical concepts or even in assuming laws, and that only a system as a whole which includes concepts, coordinating definitions, *and* laws can be said to be either true or false, to be adequate or inadequate to empirical facts. This "freedom," however, is a somewhat doubtful gift. The manifold of possibilities implies uncertainty, and such uncertainty can become rather painful in a science as young as psychology, where nearly all concepts are open and unsettled. As psychology approaches the state of a logically sound science, definitions cease to be an arbitrary matter. They become far-reaching decisions which presuppose the mastering of the conceptual problems but which have to be guided entirely by the objective facts.

Theoretical psychology in its present state must try to develop a system of concepts which shows all the characteristics of a Gestalt, in which any part depends upon every other part. As we do not yet have the knowledge of facts which really suffices to determine this system of concepts and as, on the other hand, this knowledge of "facts" cannot be acquired without developing this system of concepts, there seems to be only one way open: to proceed slowly by tentative steps, to make decisions rather reluctantly, to keep in view always the whole field of psychology, and to stay in closest contact with the actual work of psychological research.

Such an undertaking, if any, needs the cooperation of a group. I have always found myself rather unable to think productively as a single person. I hope that this handicap may, in this case, turn out to be of some advantage, for it has made this book the result of the work of a group. Those who are acquainted with you know that you are not interested in "psychological schools," and one of the main incentives of this book is to help develop a psychological language generally understandable and independent of schools. (By the way, I have tried my best to destroy the myth that Gestaltists do not attack each other.) Yet collectives have had and will, I think, always have their place in scientific work. The group which was called the Psychological Institute of Berlin has been, I think, such a collective of friends, working together for many years, interested in all fields of psychology, and concerned as much with experiments as with theories. Whether it was valuable, history will show; but at least it was happy and lively.

May this book prove to be somewhat worthy of the spirit of this collective and of the leading influence you have had on each of its steps. For the friends scattered throughout the world this feeling of cooperation seems to continue and the circle steadily to widen. I would enjoy nothing more than to have contributed to this broad cooperation.

I dedicate this book to a young scientific center at the meeting of the East and the West where I hope new productive collectives will arise.

KURT LEWIN.

IOWA CITY, IOWA,
May, 1936.

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Dr. Fritz Heider and Grace Heider have not only undertaken the laborious work of translating this book, but have improved its form and contributed much to its content. I am deeply indebted to the productive help and criticism of Dr. Tamara Dembo. She, Dr. Roger Barker, and Dr. Herbert Wright have spent much time in improving the text. Dr. W. W. Flexner was good enough to read the part dealing with the topological concepts and to give valuable suggestions. I gained much by discussing several points with Dr. Herbert Feigl, Dr. W. A. Hurwitz, Dr. E. H. Kennard, and Dr. E. C. Tolman.

Harcourt, Brace & Company has kindly permitted the use of a selection from Anne Morrow Lindbergh, *North to the Orient*. Figure 6 is taken from Charlotte Bühler, *Zwei Grundtypen von Lebensprozessen;* Fig. 7 from Kurt Koffka, *Principles of Gestalt Psychology.*

KURT LEWIN.

IOWA CITY, IOWA,
May, 1936.

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PART ONE

THE TASK OF PSYCHOLOGY AND THE FOUNDATIONS OF TOPOLOGICAL AND VECTOR PSYCHOLOGY

CHAPTER I

INTRODUCTION

THE PRESENT STATE OF PSYCHOLOGY

In its present state of development psychology must be thought of as a young science. There is only one field in which it is relatively well established and in which it has advanced steadily: this is the psychology of sensation and perception. The scientific character of this field is fully recognized. Its findings are based almost entirely on experimental evidence, and even when its theories are in conflict one feels that as far as method is concerned it stands on relatively firm ground. The situation is different with the psychology of will, of needs, and of personality despite the fact that these fields have always attracted popular interest. As recently as fifteen years ago it was assumed that they, by their very nature, were not amenable to scientific methods. The little experimental work that had been done seemed too artificial and abstract to give an insight into the real processes. It was generally accepted that experimental investigations of these elusive and highly complicated processes were intrinsically impossible, at least in so far as human beings are concerned. Thus in Europe these problems were treated in a half-literary, half-philosophical way, and in America the tendency was to study individual differences by means of tests.

The only approach to deeper problems was the brilliant work of Freud. However, the attempt of the psychoanalysts to base general laws entirely on case studies and therapeutical work seemed methodologically unsound to most scientists.

This skeptical atmosphere and the undoubtedly great technical and conceptual difficulties have blocked the development of an experimental psychology of will and needs. On the other hand a number of branches of psychology have reached a stage which makes their unification increasingly urgent. Child psychology, for instance, has collected a great number of facts about speech, play, and other forms of behavior at different age levels. Animal psychology has to a large extent passed beyond the more elementary questions and has begun to study more inclusive and in many respects more "human" problems. Psychopathology has brought together a great number of facts that ought to be directly related to facts of normal psychology. Finally problems of social psychology which lie across all these fields are becoming more and more urgent.

From all these sources we possess a great amount of valuable material. At the same time, especially in recent years, we have become much more critical of what we have done. In America there seems to be an increasing distrust of purely statistical methods, and the indiscriminate use of tests is criticized in a way which would have seemed impossible a few years ago.

Investigators are coming to feel that a mere piling up of facts can only lead to a chaotic and unproductive situation. The simple collecting of facts is indispensable at certain stages of a science; it is a wholesome reaction against a philosophical and speculative building of theories. But it cannot give a satisfactory answer to questions about causes and conditions of events. Only with the help of theories can one determine causal interrelationships. A science without theory is blind because it lacks that element which alone is able to organize facts and to give direction to research. Even from a practical point of view the mere gathering of facts has very limited value. It cannot give an answer to the question that is most important for practical purposes—namely, what must one do to obtain a desired effect in given concrete cases? To answer this question it is necessary to have a theory, but a theory which is empirical and not speculative. This means that theory and facts must be closely related to each other.

Psychology needs concepts which can be applied not merely to the facts of a single field like child psychology, animal psychology, or psychopathology, but which are equally applicable to all of them. One should be able to use the same concepts for problems of emotional life as for problems of behavior; or for problems concerning the infant, the adolescent, and the aged; the healthy and the sick; animals and human beings; the personality and the environment. Does this mean that we are to return to the making of speculative "systems"? Yes and no. Yes, in so far as we should not content ourselves with a blind collecting that splits the field of psychology into a number of unrelated branches. No, in so far as we must not try to derive all psychological facts neatly from one single concept such as association, reflex, instinct, or totality.

The system of concepts capable of bringing together the different fields of psychology in an empirical manner would have to be rich and flexible enough to do justice to the enormous differences between the various events and organisms with which it must deal. It would therefore have to be oriented in two directions, namely, toward theoretical connectedness and toward concreteness. In other words it would have to be equally suitable for the representation of general laws and of the characteristics of the individual case.

The unification of the different fields of psychology seems quite hopeless until we have an adequate psychology of will and needs and of personality. Fortunately, however, we need not feel pessimistic about the possibility of developing these central regions of psychology. Within recent years a great number of studies have shown that in spite of the general skepticism an experimental attack on fundamental problems in those fields, including problems of Freudian psychology, is quite possible.

We have come to see that in investigations of this kind we must deal with persons as wholes to a much greater extent than in the psychology of sensation. In the psychology of sensation the individual's ideals, ambitions, and his social relationships play no role at all or only a subordinate one. But an experimental investigation of needs, of action, or of emotions cannot be carried out without taking into account the characteristics of the person, his momentary state, and his psychological environment.

This shows again that the concepts of which psychology is now in need have to meet the requirements which we suggested above: the system of concepts must be broad enough to be applicable to the most primitive bodily behavior as well as to the emotions, thought processes, values, and social relationships. It must be capable of representing these processes not as single isolated facts but in their mutual dependence as expressions of a concrete situation involving a definite person in a definite condition. These concepts must unify without undue simplification; they must include both person and environment, both law and individual case.

These requirements can be fulfilled only if one turns from the prevailing methods of "abstractive classification" and tries to build constructive concepts.

The concepts which are discussed in the following chapters have been developed and tried out in the course of the last ten years. They are based on both experimental investigations and case histories. In presenting them we are not promulgating a new "system" limited to a specific content, but rather we are describing a "tool," a set of concepts by means of which one can represent psychological reality.

As I see it, the outstanding characteristics of this undertaking are:

1. It tries to build up a framework for the constructive representation and derivation of psychological processes which is logically consistent and at the same time adapted to the special properties of the "psychological life space."

2. It includes both the characteristics of the environment and of the person.

3. It makes no more assumptions than are required.

4. It proceeds by a method of successive approximation.

The concepts developed in the following pages are "operational" in so far as a univocal relation between concepts and observable data is consistently maintained. Although the concepts always extend from the level of phenomena into the level of causal relationships, they are "descriptive" in the sense of Newton's dictum: *Hypotheses non fingo*. That is, they express the nature of certain relationships and at the same time avoid that type of "explanation" which is characteristic of speculative theories and which is at present, as it seems to me, a real handicap to our science.

The concepts that we here offer will certainly have to be revised in the course of time. But I am optimistic enough to believe that they, independent of all schools of thought, will prove themselves to be fundamental to psychology in that the later additions and changes will leave the validity of these concepts, as first approximations, intact. Such a stability is after all the only one possible in science.

The purpose of the following chapters is not that of a textbook of psychology. There is no attempt to give an account of the psychological data which have been found with the methods discussed herein, or to present the specific theories developed with these concepts. I have merely tried to comply with requests to give in extenso the definition of the concepts and the mathematical background used in the research which is published under the title Untersuchungen zur Handlungs- und Affektpsychologie, I to XX, edited by Kurt Lewin in the *Psychologische Forschung*, 1926 to 1937. Concerning the fruitfulness of these concepts, one should turn to these particular studies or to Dembo and Hanfmann (19) and Lewin (51, 55, 58).

The term "topological psychology" is used to refer to that part of theoretical psychology which is based upon concepts of mathematical topology. It is to be complemented by "vector psychology." In actual research, of course, both types of concepts have to be used together.

The general, rather extensive first part of this book is an introduction to both of these fields of theoretical psychology.

CHAPTER II

FORMULATION OF LAW AND REPRESENTATION OF SITUATION

From the viewpoint of theory of science, the recent development of psychology corresponds in magnitude, extent, and character to the transition in physics from medieval Aristotelian to modern Galilean concepts.[1] It is one of those advances which are typical of certain stages in the development of sciences and which narrow the gap between a still half-speculative theory and concrete reality in a decisive way.

Law and Individual Case

One of the most striking features of this development is that the opposition between universal concept and individual event is overcome. Law and single occurrence enter into intimate relationship. Thereby, the representation of single concrete cases gains a new fundamental meaning for science. Heretofore the single event could be thought of as only a chance occurrence and its representation could be valued merely as a curiosity. Only an average of many cases seemed to possess general significance. But if one considers the single event also as governed by law, one has to obtain scientific evidence from concrete "pure cases" and not from averages of a great number of historically given events. Thereby the representation of single cases gains new scientific meaning. It has a direct bearing on the determination of general laws.

Tables 1 and 2 compare main features of three developmental epochs, which we designate briefly as "speculative," "descrip-

[1] There is a discussion of the methodological and conceptual aspects of this change in K. Lewin (57, 59). Also J. F. Brown (8, 9, 10).

The numbers in parentheses refer to the items in the Bibliography in the back of the book.

tive," and "constructive." (It may be emphasized that this comparison is a rough schematic simplification.)

TABLE 1.—CHARACTERISTICS OF CONCEPTS AND METHODS IN DIFFERENT EPOCHS OF PSYCHOLOGY

Epoch	I Speculative ("Aristotelian")	II Descriptive	III Constructive ("Galilean")
Goal	To discover the essence of things and the cause behind all occurrence	To collect as many facts as possible and to describe them exactly	To discover laws. To predict individual cases
General characteristics of concept formation	Psychological concepts are not separated from non-psychological		Elimination of non-psychological concepts
	Dividing psychology into independent fields with different laws		Psychological phenomena treated as one field governed throughout by the same system of laws
	Friendly to theories (speculative type)	Hostile to theories	Friendly to theories (empirical type)
Historical and systematic problems	Problems of occurrence and of quality are not separated		Problems of occurrence and of quality are separated
	Historical origin and cause not clearly differentiated		Historical origin and cause are differentiated
Type of system	All-inclusive system derived from a single concept or from a few dichotomic concepts	Descriptive classification by abstraction	Constructive system based on a group of interrelated concepts. Concepts which permit gradual transition between oppositions

THE CONSTRUCTIVE REPRESENTATION OF THE SITUATION

In addition one has to consider the following fact. As late as the end of the nineteenth century the question was still debated whether psychology should only describe or whether it should also try to determine the conditions and effects of psychological processes. At the present time we find that questions about the "why," or in other words dynamic questions, claim the center of interest in both theoretical and applied psychology.

In order to answer these questions it is necessary to find out the laws which control psychological events. This means that

TABLE 2.—LAWFULNESS AND DYNAMIC CONCEPTS

Epoch	I	II	III
Nature of lawfulness	A Law = a rule. Individual case not lawful. Lawfulness exists only where there is a regularity of occurrences		A law ≠ a rule. All events are lawful including those which occur only once. An empirical proof that an event is lawful is not necessary
The technique of proving a particular law	Demonstration of the frequency of similar events, disregarding individual differences. The rule is the more certain the greater the number of cases and the greater their similarity. "The exception proves the rule"		Investigation of individual "pure cases." Comparison of different cases (systematic variation); no abstraction from individual peculiarities. The validity of the proof depends upon the purity of the case and not upon the frequency of its occurrence. Experiment = deliberate creation of pure cases
Logical properties of concept formation	Classification by abstraction from differences (statistical average). The concept of thing predominates	Classification according to the phenotype	Concept formation through construction (as opposed to classification). Genetic definitions. The concept of event predominates; functional, conditional-genetic concepts
Dynamics	Causes are directed factors (tendencies). The essence (general class) of the thing itself is the cause of its behavior. The behavior is determined by the past or the future (teleology)		Causes are directed factors. Only relations between several facts can be causes of events. Every event depends upon the *totality* of the *contemporary* situation

one must determine under which conditions the different kinds of psychological events occur and what effects they have. But knowledge of the laws alone does not answer the question of why in a particular case a given individual behaves in a given way and not otherwise. Even if all the laws of psychology were known, one could make a prediction about the behavior of a man only if in addition to the laws the special nature of the particular situation were known. The laws define functional relationships between different characteristics of an event or situation. The application of the laws presupposes the comprehension of individual cases. One can apply a law only if one knows the nature of the concrete case with which one is dealing. Considered from this point of view the laws are nothing more than principles according to which the actual event may be derived from the dynamic factors of the concrete situation.

This relationship can be made clear by the following formulation: If one represents behavior or any kind of mental event by B and the whole situation including the person by S, then B may be treated as a function of S: $B = f(S)$. In this equation the function f, or better its general form, represents what one ordinarily calls a law (84, p. 366). If one substitutes for the variables in this formula the constants which are characteristic for the individual case one gets the application to the concrete situation.

The determination of the laws is therefore only one side of the task of explaining mental life. The other side, which is of equal importance and inseparably connected with the determination of the laws, involves the task of representing concrete situations in such a way that the actual event can be derived from them according to the principles which are given in the general laws. The usual description of a situation does not make this possible. It can be done only by means of a constructive representation of a situation. We shall discuss the necessary characteristics of such representations in detail.

Person and Environment; the Life Space

As far as the *content* is concerned, the transition from Aristotelian to Galilean concepts demands that we no longer seek the "cause" of events in the nature of a single isolated object, but in the relationship between an object and its surroundings. It is not thought then that the environment of the individual serves merely to facilitate or inhibit tendencies which are established once for all in the nature of the person. One can hope to understand the forces that govern behavior only if one includes in the representation the whole psychological situation.[1]

In psychology one can begin to describe the whole situation by roughly distinguishing the person (P) and his environment (E). Every psychological event depends upon the state of the person and at the same time on the environment, although their relative importance is different in different cases. Thus we can state our formula $B = f(S)$ for every psychological event as $B = f(PE)$. The experimental work of recent years shows more and more this twofold relationship in all fields of psychology. Every scientific psychology must take into account whole situations, *i.e.*, the state of both person and environment. This implies that it is necessary to find methods of representing person and environment in common terms as parts of one situation. We have no expression in psychology that includes both. For the word situation is commonly used to mean environment. In the following we shall use the term psychological life space to indicate the totality of facts which determine the behavior of an individual at a certain moment.

Ways in Which the Life Space Is Represented

At present we have no adequate scientific method for representing the psychological life space. In accord with the general methods of psychology, the study of environmental influences began with classification and statistics. For instance, the average achievement of the "only child," or of the "second child in a family of three," has been investigated by these methods. In medical case studies, one usually finds more concrete detail of the psychological environment. Thus they have given us excellent descriptions of the home environment.[2] The method of representation is partly akin to that of the

[1] For the concept of field and the history of its use in psychology, compare Koffka (47, pp. 54*f.*) and Koehler (44, pp. 320*ff.*).

[2] For example, *cf.* A. Homburger (33, p. 242).

novelist, *i.e.*, one tries to make as lifelike a picture of the situation as possible by choosing expressive words and bringing out significant traits with examples. In general, the descriptions that have been most valuable to science have not been those made by scientific methods. Where theoretical concepts have been introduced with the concrete description, they often stand out as something alien. Instead of scientific description they are nothing more than speculative interpretation.

The most complete and concrete descriptions of situations are those which writers such as Dostoevski have given us. These descriptions have attained what the statistical characterizations have most notably lacked, namely, a picture that shows in a definite way how the different facts in an individual's environment are related to each other and to the individual himself. The whole situation is presented with its specific structure. This means that the single factors of the situation are not given as characteristics which can be arbitrarily combined in a "summative" way (88, 89, 90). If psychology is to make predictions about behavior, it must try to accomplish this same task by conceptual means. In selecting methods and concepts we must use a pragmatic criterion: we have to find concepts on the basis of which predictions can be made. In other words our concepts have to represent the interrelationships of conditions. This point of view will determine the procedure of this book.

CHAPTER III

GENERAL CONSIDERATIONS ABOUT REPRESENTING LIFE SPACE

The Life Space as the Totality of Possible Events

If we are to accomplish the task of deriving the behavior of the person (in more general terms: the psychological events) from the life space, we have to characterize it as the "totality of possible events."

We shall later discuss in detail what sort of events are fundamental for the representation of the environment and of the person. Here we shall only mention that from both the theoretical and practical point of view the most important characteristics of a situation are what is possible and what is not possible for the person in this situation. Each change of the psychological situation of a person means just this—certain events are now "possible" (or "impossible") which were previously "impossible" (or "possible").

For instance, when an employee of a company is dismissed, the important change for him is that he can no longer give orders to the office boys, that he can no longer make purchases for the firm, and that all other possibilities of action which he enjoyed as a member of the firm are taken away from him. These may include the privilege of using a certain entrance to the office as well as any sort of behavior toward other persons to which the prestige of the firm gave him a right. On the other hand, he can now do many things which were not possible before. He can snub his former employer, he can read books because he has plenty of time, he can sleep late in the morning, etc.

Also, the difference between the situation shortly after the dismissal and that after a long period of unemployment can be characterized by changes in the possibilities of action. As time goes on, the lack of money makes a good meal or a journey impossible. He may still be able to dress respectably, but he can no longer afford to dress elegantly. Also he has lost the courage to go out every day looking for work.

In a similar way, the difference between the rich and the poor, between the youth, the adult, and the man who is almost too old to hold a job, is in each case fundamentally determined by a range of possibilities. The same is true for the difference between the healthy man and the sick one, between people of different educational level, and people in different political situations. A dynamic psychology has to represent the personality and the state of a person as the total of possible and not-possible ways of behaving.

Inventories and Systems of Behavior

Another fact leads to the same point of view, namely, that the situation is to be regarded as the total of possibilities. One of the chief difficulties which psychology has to face is the following: If one investigates, for instance, the causes and effects of anger, one is confronted with a great variety of responses (20, pp. 27–30), although one may start with a definite experimental set-up, which roughly at least guarantees a consistent psychological structure of the situation. There appear anger effects of very different degrees, and at the same time a great number of other kinds of behavior: substitute actions, short cuts, changes of level of aspiration, aggressive behavior, etc. It is possible to classify and describe these processes. The most one can accomplish in this way is to make a catalogue of types of behavior and to amplify and refine it. The same is true if one investigates success and failure, the boundaries of the ego, punishment, etc.

Such a collection of facts is indispensable and has its scientific value. However, the real task of scientific, especially of experimental psychology, lies beyond such a collection. It is necessary to understand why this and only this behavior occurs. In place of a catalogue which gives no reason why under given circumstances just these and no other forms of behavior occur, there must be built up a framework of concepts which does not have the arbitrary character of a mere list. This means creating a system of deduction. The different kinds of behavior that occur in a certain situation are to be understood as belonging to a coherent system of "possible" events that are in their totality an expression of the particular characteristics of this situation.

The more we succeed in determining the details of the situation in this sense, the more the actual possibilities are limited. A complete determination of the life space would show which of the possibilities, given by its general structure, will be realized at the moment.

Constructive Procedure: Summary

To summarize what we have said about the representation of person and environment:

1. The fundamental constructs which we use in representing the situation must consist of concepts from which one can derive, unambiguously, certain events as "possible," others as "not possible." Instead of classificatory concepts one has to use constructive ones which have a direct relationship to laws.

2. It should be possible to derive from such a representation all forms of behavior which actually occur. This stringency of the *derivation of the totality* of possible cases is valid not only for the behavior of the person within the situation but also for the possible changes of the person or of the situation itself.

3. Such a derivation of the totality of possibilities can only be accomplished if one proceeds from the life space *as a whole.*

4. The center of interest shifts from *objects to processes*, from states to changes of state. If the life space is a totality of possible events, then "things" that enter the situation, especially the person himself and psychological "objects," have to be characterized by their relationship to possible events.

Good and Poor Abstraction; the Method of Approximation

In addition to the reasons given before, there are the following advantages in starting with the life space as a whole in making an analysis. In psychology, as in every other science, the investigation and representation of each single case is an infinite task in itself, and one whose solution would presuppose a full knowledge of the laws of psychology (7, p. 33). In comparison with this ideal, every actual representation of a concrete case is incomplete and simplified. There are two ways of meeting this difficulty, and from the point of view of research they are very different. One method, which one can call abstracting classification, begins by taking into account important single facts and then makes classifications according to one or another of these facts. The individual peculiarities of each situation are thus disregarded. Since there are almost always several such significant facts, such a classification is usually open to attack. It is ambiguous in itself and very often vacillates between opposite characterizations.

In contrast to this the second method begins with the life space as a whole and defines its fundamental structure. The procedure in this case is not to add disconnected items but to make the original structure more specific and differentiated. This method therefore proceeds by steps from the general to the particular and thereby avoids the danger of a "wrong simplification" by abstraction. In such abstractive classification, the second step often destroys the characterization of the first. The "right simplification" implies a schematization too, but it is a procedure of "gradual approximation." The representation given in the first approximation will not be destroyed but only made more articulated by the second approximation since the whole situation is taken into account from the beginning. To what point the approximation shall be carried depends upon the particular problem under consideration. It is important that even the representations in the first approximation are of value in their own right. In this way the formation of concepts becomes essentially similar to that in mathematics.

CHAPTER IV

CONTENT AND EXTENT OF THE PSYCHOLOGICAL LIFE SPACE

What is meant by psychological life space and what must one take into consideration in order to represent it?

Appearance and Reality in Psychology

Certainly one will have to represent the physical environment of the individual to a certain extent, for instance the room where he is and the position of the furniture and other objects that are important for him at the moment; in certain cases also the house in which the room is, the city, and even the country. One will have to represent his social environment, his relationships to other persons, their positions and personalities, and his own place in society, for instance his vocation. At the same time, his longings and ambitions will play an important role, his fears, thoughts, ideals, and daydreams, in short everything that from the standpoint of the psychologist exists for this person.

Experience and Psychological Existence.—It is, however, not always easy to determine what things exist psychologically for a given person. The most obvious method might seem to be the use of consciousness as a criterion. This would mean that the physical and social environment would be treated as psychological environment in so far as the person is conscious of them. Such a formulation is doubtful, however, even if one uses the concept of consciousness in a very broad manner. There is no question, for instance, that when a person is in a familiar room, the part of the wall which is behind him belongs to his momentary environment. Furthermore, the fact that such and such other rooms are near this one, that the house stands in a lonely settlement by the sea or on a busy thoroughfare of a great city can be an essential part of the psychological situation. This can be true even when one is not looking at the landscape but is deep in his work and for the moment is not, or at least in no clear way, aware of this wider environment.

It is likewise doubtful whether one can use consciousness as the sole criterion of what belongs to the psychological life space at a given moment in regard to social facts and relationships. The mother, the father, the brothers and sisters are not to be included as real facts in the psychological situation of the child only when they are immediately present. For example, the little child playing in the garden behaves differently when he knows his mother is at home than when he knows that she is out. One cannot assume that this fact is continually in the child's consciousness. Also, a prohibition or a goal can play an essential role in the psychological situation without being clearly present in consciousness.

The same is especially true for the general social atmosphere, its friendliness, unfriendliness, or tension. Doubtless, just these general properties of the social atmosphere are of the greatest significance for man's behavior and for his development. And yet, one often realizes what the atmosphere has been only when it changes.

What Is Real Is What Has Effects.—Here, as in many other cases (57) it is clear that one must distinguish between "appearance" and the "underlying reality" in a dynamic sense. In other words, the phenomenal properties are to be distinguished from the conditional-genetic characteristics of objects and events, that is, from the properties which determine their causal relationships. From the standpoint of dynamics one must consider the whole situation as the total of what has effects for the individual under consideration. As far as the conceptual derivation is concerned, one may use effectiveness as the criterion for existence: "*What is real is what has effects.*"[1]

[1] Thus far this view is in line with that of the New Positivism (*cf.* Feigl, 24*a*, p. 422), although we are here less concerned with the problem of "the reality of mind." Our criterion rather serves as a tool for making practical decisions

Phenomenal Facts and Physics.—The distinction between phenomenal and conditional-genetic properties must not be confused with the distinction between psychological and physical objects. In psychological and philosophical discussions it is common to identify the psychological with the "directly given." Koffka (47, pp. 46*ff.*), by calling attention to unconscious processes and reflexes, has clearly demonstrated that the experienced world (behavioral environment) does not suffice to explain behavior. Nevertheless he seems to hold to the abovementioned interpretation in so far that he does not object to the identification of conditional-genetic and physical facts. According to this point of view the physical is only indirectly comprehensible;[1] it has to be inferred from psychological experience. The psychological is not limited to the phenomenal, but on the other hand everything phenomenal is something psychological. This conception has far-reaching consequences. It implies that the conditional-genetic, dynamic facts belong to the physical world, even in psychology, and therefore that explanation of psychological events in the last analysis has to be based upon physical facts.

These and similar conceptions are widely accepted. However, they seem to me erroneous, both from an epistemological and from a psychological point of view. The objects of all empirical sciences, including the objects of physics, can be experienced no less directly than those of psychology. This direct experience concerns first of all the appearance of objects, *i.e.*, their phenomenal properties. In order to understand causal relationships one has to proceed to the conditional-genetic properties. But this progression to deeper levels takes place within one and the same field of science. The conditional-genetic properties of a piece of iron that physics finds remain properties of this same piece of iron, which one perceives directly and uses, however far the concepts of physics may

in psychology. We do not presuppose, as New Positivism generally does, the reducibility of Psychobiology to Physics.

[1] He says: But every *datum* is a behavioral datum; physical reality is not a datum but a constructum (47, p. 35).

progress and however indirect physical methods may be. Otherwise the scientific analysis would be meaningless, both from a practical and from a theoretical point of view.

In the same sense one can distinguish in psychology between the properties which are more phenomenal and more directly accessible and the conditional-genetic properties of its objects.[1] In psychology as well as in other sciences an explanation of events is only possible if one succeeds in advancing to the dynamic properties. And again the phenomenal and the dynamic properties are properties of one and the same psychological event.

Such a point of view recognizes in both physics and psychology phenomenal as well as conditional-genetic properties. This is not the place to give detailed proof of our argument. Certainly it has great methodological advantages. The point of view according to which all psychological explanations must finally rest on physics is based essentially on the philosophical Utopia of a single universal science. But this means that in advancing from description to explanation psychology is forced to make a sudden jump into an alien field. If one disregards this philosophical Utopia one can represent a life space in a continuous progression and take into account all necessary dynamic facts, whether they are determined directly or indirectly. This method seems to me the only one that makes it possible to include in one representation everything which is and only that which is necessary for the conceptual derivation of actual behavior.

A thoroughly worked out dynamic representation of person and environment will have the character of a construction and it must have this "conceptual" character if it is to serve as a means of deriving actual behavior. But such a construct (10; 7, p. 3; 84) must not be confused with a general class con-

[1] Tolman (84) points to the fact that one cannot derive behavior directly from behavior. Instead, one has to introduce some "intervening" variables between the behavior to be derived and the observable facts indicative of its causes. So far as I can see, these intervening concepts are the same as our "dynamical" or "conditional-genetic" concepts. The term "intervening concept" may prove to be quite convenient.

cept because it represents the characteristics of a concrete individual case.[1]

Life Situation and Momentary Situation

By psychological situation can be understood either the general life situation or more specifically, the momentary situation.

A woman stands at the loom in a big noisy factory, next to the last in the eighth row. A thread is broken. She is about to stop the machine to see what has happened. It is shortly before the lunch hour. She has accomplished very little during the morning. She is annoyed.

These are a few data of the momentary situation of this woman. About her life situation, one can say:

She has been married for three years. For a year and a half, her husband has been unemployed. The two-year-old child has been seriously

[1] The necessity of psychological theories which permit logically strict derivations of the concrete facts is stressed by C. L. Hull (35). Hull formulates four postulates of a "sound scientific theory" (35*a*, pp. 495*f*.). These postulates concern its logical characteristics and the way a theoretical system should be proved. He stresses "that the deduction or proof of each theorem is a complex multiple-link logical construct involving the joint action of numerous principles or postulates, as contrasted with simple syllogistic reasoning where but two premises are employed" (p. 499).

We fully agree with Hull's four principles, which are in line with our own postulate 1 (pp. 6 and 16). We too want to stress particularly that no single concept suffices as a basis for derivations. But it may be well to say a word about the limits of a purely formalistic approach. Hull probably would agree that a theoretical system might be logically sound but have such a poor psychological content that its scientific value would be negligible (compare, for instance, Stevens 80*a*). Psychology needs stressing of the formalistic. Yet, it would soon prove most unfortunate if one should lose sight of the fact that the main purpose of psychological theories is, after all, to explain reality. In psychology, for a long time to come, the richness and fruitfulness of theories should not be judged by their formalistic perfection alone.

Logically it may be conceivable that in representing psychological dynamics one can use any kind of concepts. (In other words, one may disregard the possibility that there is something like a "logic of dynamics.") Practically, however, form and content of a theory are both fundamental and we mean to stress both aspects equally. For, a system which is inadequate or unclear as to the content of its dynamical concepts will soon prove to be unsatisfactory logically. It would lead us too far to discuss from this point of view Hull's "Miniature Scientific Theoretical System" (35*a*, pp. 501*ff*.).

ill, but today he seems somewhat better. She and her husband have been quarreling more and more often recently. They had a quarrel this morning. Her husband's parents have suggested that she send the child to them in the country. The woman is undecided what to do about it.

It is obvious how closely life situation and momentary situation are connected. In this case, the life situation may serve as a rather remote background of the momentary situation. Or it can be that the woman was thinking of her child while she worked, and in this way the life situation often became part of the immediate situation. But even when she was busy repairing her broken thread and no longer thought of the domestic situation, even then the life situation remained at least indirectly significant. It affected the state of the person and thereby the reactions within the momentary situation.

The woman sees the momentary environment, the rooms, the bed, the household routine, in a different light with each change in the life situation. Objects which were dear to her before the trouble with her husband might have become disagreeable, others the more precious. The room in which a child is ill changes its character and changes it once more when the child recovers. Their past history thus plays a great part in determining the psychological import of things for the person.

Although the whole life situation always has some influence on the behavior, the extent to which one must take it explicitly into account in the representation of the life space is very different in different cases. A person who is trying to decide whether or not to get married, whether or not to go into a certain business, whether or not to begin a lawsuit against an influential opponent, will in general act in accord with his whole life situation. Only happy-go-lucky, superficial, or childish persons act out of a narrow momentary situation in important questions of life. On the other hand whether a man who is taking a walk goes along the right or the left side of the road will be decided by a much less inclusive momentary situation. It is easy to observe how the structured part of the life space becomes wider or narrower under the influence of a new event.

In representing the life space it is necessary to take into account these differences of extension. The specific problem with which we have to deal in a given case determines whether it is the life situation or the momentary situation which comes more strongly into the foreground. The life situation is on the whole the more constant. In representing it one can reckon with larger time units. These differences in rate of change do not however mean that there is a fundamental difference between the two. The concepts which are developed in the following can be equally well applied to life and momentary situations, *i.e.*, to any possible life space.[1]

Quasi-physical, Quasi-social, and Quasi-conceptual Facts within the Life Space

If one uses the dynamical criterion ("what is real is what has effects") to determine the existence or nonexistence of a fact as a part of a psychological life space, one has to include a great number of facts. For example, one would have to include all somatic processes which influence the behavior of a person, for there can be no doubt that in psychology we have to deal with psychobiological organization and that psychology is only a part of the general science of life.

The Quasi-physical Facts.—This does not mean that we have to include within the psychological life space the whole physical world with its "objective" characteristics in terms of physics. These facts are to be included in the representation of the psychological life space only to the extent and in the manner in which they affect the individual in his momentary state. We express this by calling them quasi-physical facts.

Even when from the standpoint of the physicist the environment is identical or nearly identical for a child and for an adult, the psychological situation can be fundamentally different. The same is true for the environment of men and animals (60, pp. 322–323) and also for men of different personality. Further a physically identical environment can be psycho-

[1] Examples of constructive representation of the life situation are given by T. Dembo and E. Hanfmann (19); K. Lewin (58).

logically different even for the same man in different conditions, for instance when he is hungry and when he is satiated.[1] This does not mean that psychology can ignore the postulate of general validity for scientific statements. It only means that the situation must be represented in the way in which it is "real" for the individual in question, that is, as it affects him. The confusion of "objective" with "physical" and of "logically general" with "equal for all" has led to grave conceptual and methodological errors in psychology.

The Quasi-social Facts.—A distinction analogous to that between physical facts in the sense of physics and quasi-physical facts is that between objective social facts[2] in terms of sociology and social-psychological facts which have to be taken into account in representing a certain life space.

When a mother threatens an obstreperous child with the policeman and the child obeys her because of his fear of the policeman, then as far as the representation and explanation of the child's behavior are concerned we are dealing not with the actual legal or social power of the police over the child, but rather with the power of the police as the child sees it.

The same applies to the power of Santa Claus, of the father, in short, to all social relationships. As regards membership in a group we have to consider more the belief of the person and the way it affects him than legally or sociologically defined criteria of group membership. In representing the psychological situation we have to include social, like physical facts, only in so far and only in the manner in which they influence the person under consideration. On this account, we shall speak of quasi-social rather than of social facts.

[1] Allport (2), p. 178, says: "Speed apparently is another factor that is homogeneous only to physics; in our results speed seems to split into three relatively independent rates of movement. Many of the speed measures correlate more highly with non-speed measures than with each other. In short, physical categories of movement are unsuitable models of the *psychological* study of expression."

[2] It is not necessary at this point to go into the epistemological question of the "objectivity" and "reality" of social structures as sociology defines them, nor into the problem whether it is justifiable to compare this reality with the reality of physics.

The relationship between purely sociological and quasi-social facts is complicated by the knowledge that even for sociology as such the opinions of people about the social relationships in which they live play an important role. This is true, even with people who consciously have no adequate realization of their social relationships. Certainly the behavior of nations would be different if they were fully conscious of their real social interdependence. Thus the social-psychological facts (the quasi-social facts of psychology) have great significance for sociology itself and the representations of psychological, especially of social-psychological facts, might be applied fruitfully in sociology.

The Quasi-conceptual Facts.—In addition to the quasi-physical and the quasi-social facts one has to consider quasi-conceptual facts as important for the psychological life space. We do not want to assume here a conceptual realism, but one may not neglect the following functional equivalence.

A person may be engaged in solving an extensive conceptual, for instance a mathematical problem. He has to follow definite steps in his thinking to determine the suitable mathematical relationships and to find his way in a system of mathematical concepts. The structure of the psychological environment in which the individual moves about, in which he faces difficulties, and in which he carries out certain tasks is then essentially determined by the structure of the mathematical field itself. The analogy to quasi-physical and quasi-social environment holds in this respect also, that we again have to deal with "given" facts which can be more or less adequately comprehended and according to whose objective structure the individual concerned must adapt himself if he wants to attain certain goals. In this respect the mathematical facts are sometimes dynamically not less cogent and insurmountable than certain physical and social facts.

Again there is not usually a complete agreement between the objective structure of the mathematical field and the momentary psychological field. If there were such complete correspondence, then the solution of mathematical problems would be child's play. Rather, the psychological field is usually incomplete in comparison with the structure of the mathematical task, and is inadequate in decisive points. For the derivation of the actual psychological events and their dynamics it is not the mathematical system in itself which is important, but rather the momentary structure of the psychological field of the individual. Therefore we shall speak of "quasi-conceptual" facts.

Finally, it may be mentioned that the quasi-physical, quasi-social, and quasi-conceptual facts are not sharply separated from each other, but that we are dealing throughout with a unified *psychological life space* in which these three groups of facts can be thought of as representing three only roughly distinguishable classes.

For the child with his animistic and magic thinking these differences remain fluid; with adults also there are many transitions and many facts which can be classified in these groups only with difficulty.

Influences by Way of Perception and "Gross Somatic" Influences

The quasi-physical and quasi-social facts in the psychological life space need not be an adequate representation of the objective physical and social facts to which they refer. However, the structure of these psychological facts depends to a high degree upon the structure of the physical and social facts. A change in the quasi-physical facts in the life space of the person is often the result of an objective change in the physical environment.

One can roughly distinguish two cases in which the life space is influenced from the outside: (1) The influence can occur by way of a perceptual process, usually leading to a change of the cognitive structure (85) of the field with reference to the object in question. (2) The influence can be a gross somatic one. A stone may hit a person and cause injury or loss of consciousness. This stone need not necessarily appear in the perceptual field of the person.

It may be questionable whether such gross somatic influences ought to be treated by psychology. Certainly the perception of a physical object and an injury inflicted by a stone are events of very different character. But the effect of a perception also may go beyond a change of the cognitive structure of the life space. It may, for instance, produce a change of the goal and lead to a change in the person's direction of action. On the other hand a gross somatic influence, for example poisoning, can also involve far-reaching cognitive changes in the life space. Perceptual processes as well as gross somatic influences can therefore change the life space in every respect. One must remember that the transition from the grossly somatic effects of physical objects to their effects as objects of perception is not at all an abrupt one.

This is especially clear in regard to the field of action. When, for instance, a man is moving a heavy piece of lumber and is pushed to one side or lifted up by it, when he swims, when he goes up steps or moves in any other way—then the changes effected by physical objects which occur in the environment and in the state of the person are not the result of perceptual processes alone. Moreover, we find even within the psychology of perception a transition and interplay of the two different kinds of effects. For instance, looking at a too-intense light may result in an injury to the eye. Similarly atmospheric conditions affect a person not only through perception but also in other ways.

In the following, we shall deal with the purely "somatic" influences only in so far as they are connected with behavior and perception. But we want to emphasize that this distinction is not a fundamental one and that it is not possible for psychology to disregard entirely the gross somatic influences. Probably there is a certain dynamical similarity between all these influences of the physical world on the life space in so far as they force certain changes upon the individual.

Likewise sociological facts need not necessarily influence the psychological life space of the person by means of perceptual processes. An arrest, a change of one's legal position by a new law, or the coming of a new employer can act from the outside, like a flying stone, and alter fundamentally the situation of an individual. This alteration need not be the result of the dynamics of the previous psychological situation.

What concepts psychology can use in dealing with such influences of "nonpsychological" factors is a difficult question which we shall treat later. In any case the extent of such influences on perception and action makes it impossible to disregard them in psychology. This means that it should be possible in principle to represent all bodily changes of the person in the life space. At the same time it confirms our position that psychology and biology are not essentially separated but are distinguished only by a difference of interest.

CHAPTER V

CAUSAL INTERCONNECTIONS IN PSYCHOLOGY

The Historical and the Systematic Concept of Causality

One derives psychological events by tracing them back to the dynamic relation in which they have their source. This "tracing back" and the concept of causation which is implied in doing so are understood in very different ways in psychology.

The question "why" can have two very different meanings in psychology.

1. Why does a given situation S (*i.e.*, a particular person P in a particular environment E) have the event B and no other as a result? As stated above this question is answered if we succeed in discovering the general law $B = f(PE)$ which is valid for the dynamic structure of the situation in question. Thus the event is traced back to the dynamic characteristics of the momentary situation. The "cause" of the event consists in the properties of the momentary life space or of certain integral parts of it.

2. Why does just such a situation come into being—*i.e.*, why has the life space in a particular case these particular properties? This question has a specific historical, or as one can say more accurately a "historical-geographical" meaning (60, pp. 328–329). It deals with historical developments, with chains of causes, and with the point of convergence of these chains. The answer to this question is obtained only by an analysis of the history of the individual and of his environment. We shall speak therefore of "historical concepts of causation" in these cases in contrast to the "systematic concepts of causation" which were characterized above.

A physical example: I am sitting in the rain under a tree whose leaves keep me from getting wet. I ask: "Why don't I get wet?" It is possible to answer this question by finding out the direction and velocity of the falling drops, the position of the leaves, my own position, etc. In short one can represent the present situation and, by applying the laws of mechanics or other relevant laws, derive what event must occur in such a situation. But the answer to this question could also be as follows: "It is thanks to your grandfather who planted this tree that you do not get wet. To be sure the soil is not very good right here, but your grandfather took special care of the tree during its first years. Yet if the plan for a new state road had gone through last year the tree would already have been cut down and you couldn't sit here without getting wet."

The second answer is an example of explanation in terms of historical causality. Its characteristic feature is that it gives an account of the course and interweavings of causal chains of events, events which happened but once and which can be placed in certain years and certain geographical locations. The answer in terms of systematic causality on the other hand refers to types and to laws in which there are no dates. Both answers are entirely legitimate and important, but neither is a substitute for the other, although one has bearing on the other. For psychology both types of answer are important. The historical concept plays an especially important role in developmental psychology, both of individuals and of groups. It is also important in psychopathology where it may be essential to understand the genesis of the illness.

Even in dealing with systematic questions in experimental psychology certain "historical" facts must be given special consideration. As the psychology of will and affect has shown (20, p. 30), the decision of some of the most fundamental problems of experimental psychology requires the setting up of unambiguous situations and this is possible as a rule only by producing a certain historical development.

In spite of the close connection between the historical and the systematic concepts of causation in research, one ought to distinguish the two much more sharply than is done at present. The confusion of historical and systematic concepts and problems is an essential characteristic of the pre-Galilean or Aristotelian thinking[1] of a period of psychology which is now coming to an end and which has led to momentous errors. It is one of the reasons for the inconsistencies of the association theory and the difficulties involved in applying the concept of experience. In psychoanalysis, whose contribution lies in great part in its emphasis on the historical aspect of its problems, it has led to important misinterpretations. Analogous conceptual confusions frequently occurred in experimental child psychology.

Only when we distinguish sharply in all branches of psychology between the two concepts of causation and in this respect also proceed from "Aristotelian" to "Galilean" ways of thinking, can we attack dynamic problems.

The following discussion will be based on the systematic concept of causation, for in dynamics one must undoubtedly give first place to this concept. Even questions of the dynamics of historical sequences cannot be answered without insight into the dependence of the single event on the given situation, *i.e.*, without determining the equation $B = f(PE)$. This does not imply a neglect or underestimation of the historical problems in psychology.

Existence, Temporal and Causal Relationships

In deriving an event from the life space, one has to take into account several principles that are important for research. They are expressed in part by the formula $B = f(PE)$, but one must emphasize them especially because present-day psychology so often ignores them.

The Principle of "Concreteness."—Only what is concrete can have effects. This proposition may seem obvious. But one often ignores it in explaining an event by development, by adaptation, by the *Prinzip der wirkenden Seele* (12), by an abstract drive, and in treating these principles as concrete

[1] *Cf.* K. Lewin (59). The same confusion of the two kinds of concepts has been no less of a handicap in other early periods, *e.g.*, of economics and history of art.

causes. It would lead too far to deal with all the consequences of such reasoning. These fallacies arise in part from a confusion between the law that governs the effects of certain concrete events and these events themselves. Effects can be produced only by what is "concrete," *i.e.*, by something that has the position of an individual fact which exists at a certain moment; a fact which makes up a real part of the life space and which can be given a definite place in the representation of the psychological situation. All this is not true of "principles."

The Relational Character of Causal Facts.—It was characteristic of the Aristotelian way of thinking (59, pp. 28–30) to derive an event from the nature of a single object, for instance, from the personality of an individual as such, from an inner drive, from an emotion. The question whether heredity or environment plays the greater part also belongs to this kind of thinking. The transition to the Galilean thinking involved a recognition of the general validity of the thesis: An event is always the result of the interaction of several facts.

This principle has a certain connection with a thesis which gestalt theory has done much to develop in the psychology of perception, namely, that the effect of a stimulus depends in part upon the nature of the surrounding field. This principle has penetrated more and more into other fields of psychology during recent years (47; 44).

If one is to derive events from forces, one will have to recognize that a force is always the result of an interaction of several facts. We shall repeatedly come across this principle and its implications.

The Principle of "Contemporaneity."—The questions of the temporal relationship of the event and the dynamic conditions which produce it are very important and have a direct bearing on almost all psychological problems. By referring to our formula $B = f(PE)$ we can state these questions more precisely: What is the temporal relationship of behavior (B) to the two factors which make up the situation, person (P) and environment (E)? Furthermore, what is the temporal relationship between the different parts of the life space?

These questions have usually been discussed in such a way that one asked whether only past or whether future events also could cause change. Wundt, for instance, believed that the characteristic of the *causa finalis* (teleology) lies in the assumption that future events influence present events. In the case of the ordinary cause (*causa efficiens*) one generally takes for granted that something past is the cause of present events. This point of view occurs frequently in philosophical discussions that are based on physics.

This emphasis on past or future causes plays an important part, not only in philosophical discussions of psychology, but also—and that is more significant for us—in the actual construction of theories regarding concrete problems. The reference to future events occurs in more or less explicit form, for instance in the application of the concepts of drive or instinct, in theories of play, etc. The causal derivation of present events from past facts plays a great role in the theory of expression, emotion, and experience. The nature of this procedure is but poorly concealed if one connects past experiences with the present by a bridge of memory. Very often the assumption of such a connection is made merely because one has followed this principle of explanation, for instance, when one bases the universality of expression of certain emotions on phylogenetically identical, formerly useful forms of behavior instead of deriving the similarity of expression directly from the similarity of the situation involved.

Though we are justified in setting up "historical" questions and looking for causal sequences, yet we must be careful to avoid historical or half-historical answers to "systematic" questions of causation. It was typical of the Aristotelian way of thinking not to distinguish sufficiently between historical and systematic questions. The result was that one took past or future facts as causes of present events. In opposition to this assumption we shall here strongly defend the thesis that neither past nor future psychological facts but only the present situation can influence present events. This thesis is a direct consequence of the principle that only what exists concretely can have effects. Since neither the past nor the future exists at the present moment it cannot have effects at the present. In representing the life space therefore we take into account only what is contemporary.

This implies that in answering questions in regard to "systematic" causation one has to represent courses of events as series of situations (momentary sections). If in the following discussion we occasionally use a diagram of a path to indicate a movement in a situation, it is only a symbolical representation and means that the structure of the situation remains sufficiently constant during the interval of time in question.

We do not have to discuss the problems of contemporaneity at this point. In order to determine the contemporaneity for different points of the field with absolute exactness one ought to take into account in psychology, as is done in modern physics, the velocity of the mediating processes.[1] But we can neglect this question for the present. It is much more important to stress the necessity of excluding events which roughly speaking belong to past and future time.

To be precise one will have to treat the single time sections not as moments without extension but as differential time sections in order to be able to determine direction and velocity of changes at given points. The concept of momentary section is to be understood in this sense in the following discussion.

One could argue that psychological facts are intrinsically of a historical nature. As a matter of fact the structure of the person and the psychological characteristics of the environment at each moment and in each point are in a decisive way dependent upon the previous history, as experimental investigations show impressively (34). However, this influence of the previous history is to be thought of as indirect in dynamic psychology: From the point of view of systematic causation, past events cannot influence present events. Past events can only have a position in the historical causal chains whose interweavings create the present situation. This fact has often not been given enough consideration in psychology.

[1] A consequence of this is that the field of contemporaneous events must be defined differently according to the point of reference in each case (74).

Historical processes in psychology as in other sciences are to be thought of as dialectical. Whether we look at short intervals or at longer stretches of time we find that periods of apparently continuous transformations are followed by periods of crisis with sudden changes of structure.

Past and Future; the Unreal and the Indeterminate in the Life Space

The relationship of past and future to the momentary life space involves several conceptually and practically important questions which physics does not have to consider.

Existence and Temporal Determination of a Psychological Fact and Its Content.—We shall begin with an example from Stendahl's *Rouge et Noir* (82, pp. 104–106):

Julien, the tutor, had resolved to touch the hand of Mme. de Rênal, his pupil's mother:

> Julien, his mind intent on his proposed enterprise, could think of nothing to say. The conversation languished.
>
> "I wonder if this is the way I shall behave on the occasion of my first duel?" the young man asked himself; for he had too much distrust of himself and others not to be aware of the mental condition he was in.
>
> Any danger would have seemed preferable to him in his mortal agony. How he prayed that Mme. de Rênal might think of some forgotten domestic duty and return to the house! The restraint that he was obliged to put on himself produced an appreciable alteration in his voice; Mme. de Rênal's voice, too, was beginning to tremble, but Julien did not notice it; the conflict between duty and timidity was too severe to admit of his thinking of anything outside himself. The great clock of the château struck the third quarter past nine, and he had not had courage to attempt anything. Disgusted with his pusillanimity, he said to himself, "When that clock strikes ten I will do what I have been promising myself all day to do, or I will go up to my room and put a bullet in my brain."
>
> After a period of suspense and anxiety—it seemed a century—during which Julien in his tense emotion thought his reason must desert him, the clock above his head struck ten. Each stroke reechoed in his bosom as if the hammer had fallen on his heart.
>
> Finally, . . .

Such an example shows that there are cases in which a future event has a strong influence on behavior, on the train of thought, on the mood. But are we really dealing with the future in these cases? If a child tries to get to a box of blocks that is visible but difficult to reach, the goal certainly is psychologically present. It is obvious that for Julien the goal, to touch the hand of Mme. de Rênal, is in the same sense part of the present life space.[1]

The following considerations offer a solution of this dilemma. The goal as a psychological fact undoubtedly lies in the present. It really exists at the moment and makes up an essential part of the momentary life space. On the other hand the "content" of the goal, ~~the touching of the hand,~~ lies as a physical or social fact in the future. Indeed it may not occur at all. The nature of the expectation and the character of what is expected, in so far as they act as psychological conditions at the moment, naturally do not depend upon whether or not the event comes to pass.[2] In either case the person strives toward a goal which exists psychologically in the present life space.

In the example given above the time relationship is as follows: The desire to touch the hand of Mme. de Rênal existed already that morning. If one were to represent the life space for a given moment of the morning one would have to include the existence of this goal as a psychologically real fact. The content of the goal is a future event, an action which is to be carried out by Julien himself in the evening. This time index of the content naturally does not determine the temporal position of the situation in which this content is included as a psychological fact. It is a qualitatively very important characteristic of this fact that the content refers to an event on the evening of the same day. This gives the goal a certain degree of nearness.

When the sun sets the intention will still persist and it will have the same content that it had in the morning. But even if it has the same content the dynamic characteristics of this intention are essentially changed, especially because of the greater temporal proximity to the goal. Not only does the goal attract Julien, but at the same time he is afraid of it. The situation is now one of grave conflict. Moreover the goal is, in another sense than in the morning, continually present.

[1] One often says in these cases: "The event is present as an image." Such a characterization is usually misleading, for there is often no real image of the goal.

[2] This shows the fallacy of the formulation that in these cases a future event acts as cause. The "future event" often never becomes a real, existing fact that can produce effects of its own.

The dynamically important characteristics of the intention change again when the time of carrying it out is fixed, and its content in this way acquires a precise time index.

We find a similar difference in time index of psychological fact and its content in the memory of, the flight from, or embarrassment over something that happened in the past. Again, the psychological reality of such feelings as fear, hope, or doubt does not depend upon whether or not the content of these feelings exists in a physical or social sense.

We can make the general formulation:

The existence or nonexistence and the time index of a psychological fact are independent of the existence or nonexistence and time index of the fact to which its content refers.

Content as a Property.—Nevertheless, the content is in no way irrelevant, but is of greatest importance for psychological dynamics. Whether, for instance, an actual goal refers to a present or a future event, whether this event is thought of as something that definitely exists, or as something that is only possible or highly improbable—all this forms an essential characteristic of a goal. Differences in time index and in existential characteristics of the content imply a qualitative difference in the psychological facts themselves, that is, they have formally the position of properties of the psychological facts.

On the other hand, in psychology as in all other sciences the time index and likewise existence or nonexistence of the facts themselves (for instance the difference between possible and real) have not the position of properties. They do not make it possible to distinguish between qualitatively different types of psychological facts.

It is an important problem how one ought to characterize the qualitative difference between these facts and whether, for instance, facts which refer to future events can always be represented within the present life space in the same way.[1]

[1] One could think of representing future and past as the margin of the life space, but on the whole such a representation does not seem permissible to the author.

The meaning of past and future in a life space is different in different cases. It is an important developmental fact that the temporal extent of the life space of a child generally increases with increasing age.

The Indeterminate.—Connected with the problem of the time index and of existence is the difficult question of determinateness and indeterminateness of psychological facts. This question is very important for the representation of situations.

In one respect our example of Julien Sorel is not typical. His goal is unusually definite, clear, and compact. When he decides to carry out the intended action at exactly ten o'clock the goal loses its last trace of indeterminateness.

Usually, goals are much less tangible, less clear and definite. The vocational goal of the fourteen-year-old may be entirely vague and uncertain. An expectation also can be very indefinite as to its content. One may feel vaguely that something is going to happen. Then it may become clearer from what direction the trouble is to be expected. Gradually one becomes aware of its real nature. In every region of mental life one finds every possible transition between the greatest indeterminateness and full determinateness.

One must emphasize that the degree of determinateness or indeterminateness of a goal, of an expectation, of a thought is in each case a dynamically important fact, and that any change of the degree of determination is a real psychological process.

It is typical of the process of orientation in a new environment that the regions which are at first unclear gradually become clearer. The degree of clearness is an essential determinant of the cognitive structure of the life space. It is closely related to the degree to which one can differentiate the life space into different regions and is therefore of great importance for learning and insight. It is essential for all situations in which practical tasks are to be completed. The definiteness or indefiniteness of a situation plays a great role in the making of decisions, in the general firmness of an individual's behavior, in his inclination to enter into a fight (18), or his aptitude for leadership (38, pp. 44–59). We are therefore dealing with important characteristics of every situation.

Indeterminateness leads to special difficulties in representing the life space. How shall one represent something that in itself is indefinite? How, for instance, can one speak of the direction toward an indefinite goal? Does not this indeterminateness involve something which in the long run makes a conceptual representation impossible?

G. E. Müller in his polemic against the concept of an image of an indefinite gray has rightly emphasized (67, pp. 425*f.*) that it belongs to the nature of a fact to have no indeterminate properties. In truth the fundamental scientific methods of proof rest on the proposition that every reality is univocally determined.

Thus we are faced with the conceptual difficulty that on the one hand each fact as such can have only definitely determined properties, while on the other hand there exists the possibility of an indeterminate goal, thought or image. I see the solution of this difficulty in that the indeterminateness of mental events is an indeterminateness of the content and not of the psychological fact itself. Fear in the face of an uncertainty that dominates a man at a certain moment is in itself a perfectly definite psychological fact whose properties it is possible to describe. One of these uniquely determined properties is the specific degree of indeterminateness of what the man fears.

Like the time index so the indeterminateness or the particular degree of determinateness of the content is to be thought of as a property of the psychological fact itself. This property is in each case uniquely determined as are all other properties of the psychological fact.

How these properties are to be represented in the life space remains a difficult and important question. We shall discuss this point later. But one must not evade the issue because of philosophical prejudices nor be disturbed by them.

CHAPTER VI

THE PSYCHOLOGICAL LIFE SPACE AS SPACE IN THE SENSE OF MATHEMATICS

Examples of Space-like Relationships in the Life Space

Whenever a life space is to be represented one tries to show that a man of such and such a nature is in such and such a situation:

John is on a ski trip in the mountains.

Janek is stable boy for the coachman of Sameschkin.

Hedwig is waiting at the police station for her passport to be issued.

Henry has been running around for three days as though he were out of his head. He had been sure that he would get the job. Now he has lost his chance.

Paul refuses all invitations. He is completely buried in his work. He sits at his desk hearing and seeing nothing.

Jack is in the dentist's chair. His eyes are closed and he is trying to forget that the dentist behind him is preparing the needle for an injection.

Small Joan is standing at a fence dreaming how the first man came down from heaven in a very, very tiny ball.

All representations of psychological life space are based on the fundamental conception of a particular person in a particular environment. Doubts begin to arise when one asks what is to be treated as person and what as environment, whether for instance clothes belong to person or environment. But however the details of the boundary between environment and person are worked out, the conception of a person in an environment is one of relative "position." The relationship "person-environment" and also the concept of "belongingness to" person or to environment imply categories which in some sense can be characterized as spacial.

Besides the person there is "in" this life space a great number of other quasi-social, quasi-physical, and quasi-conceptual facts. These facts have a certain definite spacial relationship. The life space is articulated into "regions" that are qualitatively different from each other and that are separated by more or less pervious "boundaries." In our example of the factory worker (see p. 22) the loom belongs definitely to factory and occupation. The lunch box may be characterized by a continuous change between home and factory. Many experiments have shown how important it is in which way the different regions are "connected" and in what way and to what degree they are "separated."

I am convinced that these concepts which we use for the representation of psychological facts, like region, spacial relationship in life space, connectedness and separateness, belongingness, etc., are real spacial concepts in a strict mathematical sense. It is very important for psychology to use these concepts in a strict and consistent way. Before we discuss the question of whether these are really spacial concepts we shall give a few examples of the way they have been used in our psychological investigations.

Space of Free Movement.

Example 1. We start with a simple example. Two six-year-old boys are sitting in a bathtub, the one very lively, excited, and overactive, the other quieter. The excited one (*A*) jumps around in the tub so much that the other (*B*) feels cramped. Finally *B* draws a line in the water across the middle of the tub and tells *A* to stay within his own region. Whereas in the beginning (Fig. 1*a*) there was a single unarticulated region of possible movement for *A* with the result that for the other child (*B*) the actual freedom of movement was very much restricted, now there are adjacent but sharply separated regions of free movement for *A* and for *B* (Fig. 1*b*).

Example 2. An example of a very limited space of free movement is the life space of a prisoner. His freedom of bodily movement is limited to the area of his cell. At certain hours there may be added to this the corridor and perhaps a work-room and the courtyard. This represents his whole space of free bodily movement. The region of things outside the prison that includes his family, his friends, his business is unattainable to him.

One could raise the objection that the region outside of the prison is not in every sense out of reach for the prisoner. It is

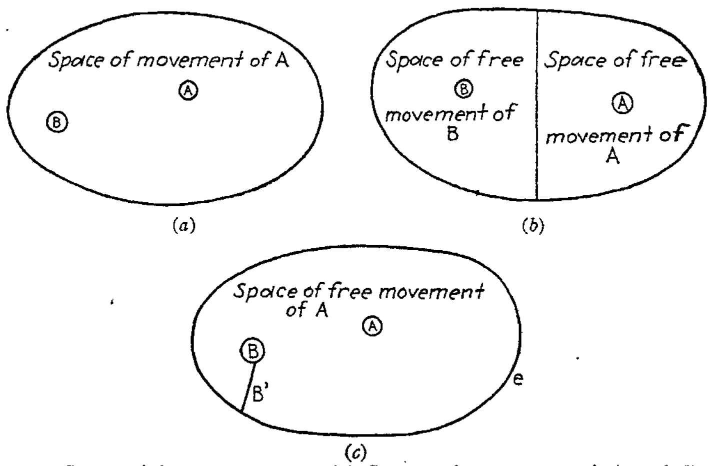


FIG. 1.—Space of free movement. (*a*) Spaces of movement of *A* and *B* are not separated; (*b*) they are separated; (*c*) see text, p. 106.

possible for him to have at least occasional "social communication" by letter and other channels with his friends and family. Further, he is able to take real action in a social sense in the outer field, for instance, by making use of his remaining legal rights.

One could ask whether there is any psychological boundary between the inner region (the prison) and the outer region, since the prisoner can, at least in thought, concern himself with objects and affairs outside the prison. This difficulty in determining the boundary of the space of free movement disappears as soon as one sees that it is only possible to determine a region univocally if one refers to specific psychological processes.

We started with a consideration of the bodily locomotion of the prisoner and the region of which we spoke referred only to this particular kind of movement. For social locomotion the structure and spacial relationships of regions in the life space generally have to be characterized in a way different from that for bodily locomotion. It is again different for mental locomotion. This difference is a manifestation of a principle which we have mentioned before and which is very important. In order to represent the life space in such a way as to answer dynamic questions it is necessary to use processes (operations) as the ultimate elements of construction.

It is characteristic of the life space of a prisoner that his space of free movement has especially rigid and almost impassable boundaries. It is dangerous and often impossible to attempt to cross these boundaries. In this connection one can speak of the solidity of a boundary. The solidity of the boundary of the prison is different for bodily, for social, and for mental locomotions.

Example 3. Two facts are especially important in regard to the space of free movement of a child: the nature and the extent of (1) what is allowed to him, and (2) what his own abilities permit him to do.

With reference to (1): A child (*C*) may be forbidden to go on the street alone, to read certain books, to go into the pantry, to climb a fence, to pick flowers, or to be impolite to guests. If within the life space of the child one outlines the region of the forbidden (*f*) then the remaining space, *i.e.*, the space of what is allowed, is for some children relatively small (Fig. 2*a*), for others considerably larger (Fig. 2*b*). This difference is very important for the behavior and development of the child, especially for his independence and his personality.

In addition to the extent of these regions the solidity and definiteness of the boundaries between the forbidden and the permitted play an important role. An adult when exerting strong control over a child can establish clear and solid boundaries between the forbidden and the permitted, or he may make the region of the forbidden very extensive, but without such solid boundaries. The result is quite different for the child in these two cases.

In this example the boundaries do not consist of physical walls as in the example of the prison,[1] but they are usually of a social nature. Their solidity depends on the character of the underlying social relationships, *i.e.*, on authority, affection, fear of punishment, etc.

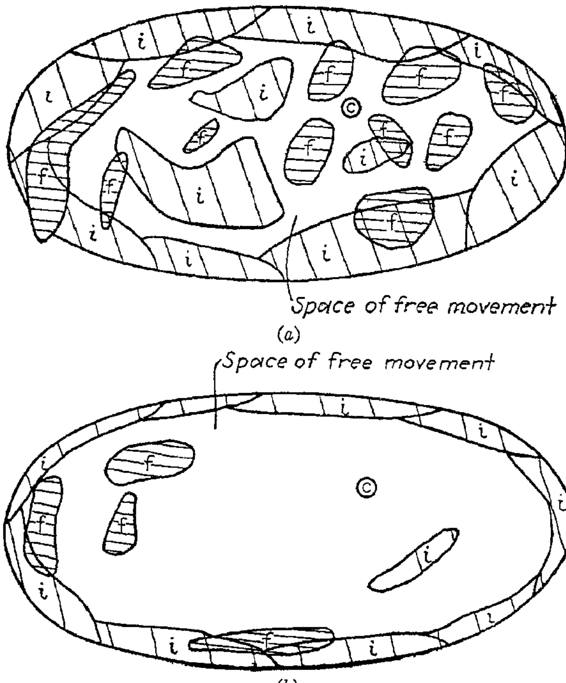


FIG. 2.—Differences in space of free movement. (*a*) Less able child in a situation with many taboos; (*b*) capable child in a situation with few taboos; *C*, child; *f*, forbidden region; *i*, region inaccessible because of insufficient ability.

With reference to (2): The freedom of movement is further limited by the fact that many goals which of themselves are permitted cannot be reached because of insufficient mental or physical ability (Fig. 2, regions *i*). The region of what is possible is in many respects much smaller for children than for

[1] The solidity of the boundaries in the example of the prison is also often based on social facts, for instance, on the legal prohibition against leaving the jail.

adults. The gradual extension of this region is one of the most important aspects of child development.

Between the gifted and the less gifted child there are great differences in this respect. For the mentally retarded child who continually meets failure and who is stopped by barriers which other children of his own age are able to pass, the space of free movement is far more limited (Fig. 2*a*) than for the superior child (Fig. 2*b*). The enlargement of his space of free movement is further hampered by the fact that through fear of failure, he holds back from attempting to enter regions which have heretofore been closed to him. Since his space of free movement is thus so greatly limited one has to be especially careful not to restrict it further by unnecessary prohibition.

Example 4. The space of movement in the social field is similarly determined by the region of the prohibited and the region of the not possible. There is a difference in freedom of movement between different classes and conditions even where they are legally equal. The rich man generally has much greater freedom of movement because of his means. He can take a special train or an airplane in order to reach his destination quickly. The poor man may have legally exactly the same rights as the wealthy one, but what is much more important for him is the fact that his social dependence and the task of supplying himself with the immediate necessities of life, such as his daily food, narrow down his freedom of movement to a much greater extent.[1] However one must not forget that we are dealing with freedom of movement in psychological space and that in certain situations the freedom of movement can be exceedingly small for any man. In any case the difference in space of free movement between persons of different social classes leads to important differences in behavior.

One of the most important goals of domestic and foreign politics is to change the space of movement of a single person

[1] Hans Fallada, in *Little Man What Now?* (24), describes very convincingly these dependencies and how they practically destroy a man's freedom of movement.

or of a group. At the same time it is one of the essential means of reaching a political goal. Political struggles as well as struggles between individuals are nearly always struggles over the boundary of the space of free movement.

Example 5 (from a motion picture). A mother has taken a year-old child away from play and wants to feed him on her lap. He does not want to eat. He is at the moment dominated by the tendency "away from eating" or "toward play." The mother holds the child on her lap and prevents the intended movement "away from eating." She puts her arm around him so that he cannot break away. The mother's interference has

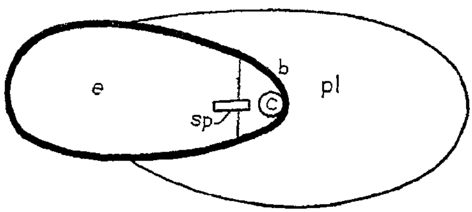


FIG. 3.—Topology of an eating situation: a child is prohibited from leaving for play. *C*, child; *b*, barrier (mother's interference); *e*, region of eating; *sp*, spoon; *pl*, region of play.

in this case the character of a barrier (*b*, Fig. 3) between the region of eating (*e*) and that of play (*pl*). This barrier at the same time keeps the child (*C*) from pulling away from the spoon (*sp*) as it is brought near his mouth. The child now begins to play on the mother's lap. The mother tries to put an end even to this possibility of action and limits the child's space of free movement still further. Thereupon the child tries to widen his region of free movement and begins to struggle with the mother.

Locomotion; Forces.—While locomotion is not the only change in the psychological life space, it is one of the most important of them. There are very different kinds of locomotion. The person himself can move about. He can approach a goal or flee from another person or from an event. This movement can take place in a direct path or by way of a detour. The movement can be free and open or secret, reluctant, and timid. It can take place rapidly or slowly. All this is true of bodily movement in quasi-physical or quasi-social fields. At the same time there is real movement in the psychological field that one cannot call bodily movement.

Example 6. The vocational goal of a sixteen-year-old boy (*P*) is to become a physician (Fig. 4). The "path" to this goal (*G*) leads through definite stages: college-entrance examinations (*ce*), college (*c*), medical school (*m*), internship (*i*), establishing a practice (*pr*). The boy may have a fairly clear idea of college. Medical school and the following stages may constitute a more or less undifferentiated region "beyond" which lies the goal of being a physician. Of this the boy may have a false but nevertheless a clear picture.

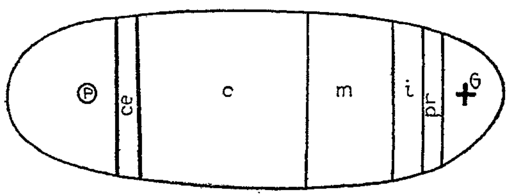


FIG. 4.—Situation of a boy who wants to become a physician. *P*, person; *G*, goal; *ce*, college entrance examinations; *c*, college; *m*, medical school; *i*, internship; *pr*, establishing a practice.

When he passes his college entrance examinations he has made a "step forward" on the way to his goal. This movement is certainly not a bodily one. Nevertheless it is real locomotion, a real change of position in the quasi-social (and as a matter of fact also in the objective social) life space. The examinations have brought him a step closer to his goal. The reality of the change in his position becomes clear when one considers that many things are now within his reach which were not before. He can go to college or university, his time is much more within his own control than before. His social position too is changed: he can play on the college football team, go to the dances, etc. His examinations therefore had for him the character of a boundary between two distinct regions. He had to cross this boundary if he wished to go from the one region to the other.

Had he failed in his examinations, then he would not have made this advance toward his goal. But also in that case there would have been a real change in his life situation. The failure would have changed the barrier between him and the region of college, which was shortly before in his immediate neighborhood. The barrier would seem much more solid, almost impassable. The youth would be thrown back and possibly would seek an entirely new goal.

To "come closer" to another person through a conversation is another case of social locomotion which, although it involves no physical movement, is psychologically real; it is the same when one withdraws from a person or a group by a particular action, or when one evades a social difficulty.

There can also be psychologically real locomotion in quasi-conceptual fields. One begins a mathematical problem. At first it presents itself as an unclear, unstructured region. Finally one may be fortunate enough to approach the solution of the problem by a particular path. This progress in thinking is a psychologically real event. The same path can be taken several times and such repetitions are different from giving up one path and trying another.

By observing the psychological locomotions one finds that there are not only thing-like objects in the life space, but also "spaces between them," *i.e.*, regions which are empty or pervious in the sense that one can move through them or along them as though they were passageways or roads. The dynamic characteristics of regions in respect to possible locomotion—especially the resistance they offer—are psychologically very important.

Not only the person himself can move about within the psychological life space but also many of the quasi-physical, quasi-social, and quasi-conceptual objects, for instance, other persons, animals, objects of all kinds. A social group may move, the power field of a person may enlarge. Whole regions can gradually or suddenly assume new positions in the life space. It often happens that someone, without really moving himself, is carried by a social movement to a certain place, or that he is pushed farther and farther from his goal against his own efforts. In another place we will describe a case in which a failure causes such a "shifting of the ground under the feet." Further, it is very important whether the life space as a whole is relatively stable, or whether it is labile and shifting.

It is a characteristic of many psychological locomotions that they are "directed" toward a certain goal or away from a certain region. If such a locomotion is stopped by an impassable barrier there can exist a tendency or, as we can also say, a "force" in the direction toward this goal. In some cases this direction is clear and definite and can easily be represented. For instance, in our examples, the boy has a strong urge to take the examinations; for the child there is a force away from the lap of the mother; the prisoner has a strong tendency to leave the prison. In other cases it is more difficult to determine unambiguously the direction of the psychological forces. A conflict may drive a person in two or more different directions at the same time. There may be a general state of tension although the directions of the tendencies are not clearly evident.

Regions within the Person.—The use of space-like concepts has likewise proved unavoidable for the psychological representation of the person. Dynamically the person appears as a "stratified" system which has a definite structure and in which one can distinguish central and peripheral regions. It has been shown that it is of great significance for problems of decision and intention, for questions of memory, of psychological satiation, of substitute satisfaction, and of emotion, whether the corresponding processes belong more to "peripheral" or to "central" regions. Furthermore it is important whether certain systems and regions within the person are "connected" or "separated." It is also relevant whether a system develops "within" this or that region of the person, and in what way the single systems go together as "parts" of a larger system. It is important whether a larger system, for instance the whole person, is strongly or less strongly differentiated into part systems. Changes in the connectedness of the systems within the person are real and demonstrable. They have, as experimental investigations have shown (92, p. 64), very important consequences.

Summary.—In attempting to characterize the psychological life space and events that occur in it one finds again and again relationships of a specific spacial character.[1]

This is especially true of the fundamental conception of the life space which implies a person "in" a surrounding field. It is true in the same way of the relationship between the parts of the environment.

The representation of the life space has to indicate the "position" of persons and objects in certain "regions." It has to take into account locomotions of a quasi-physical, quasi-social, and quasi-conceptual nature; neighborhood relationships of regions; boundaries; approaches and withdrawals; expansion and contraction; and finally movements and forces in certain directions. Whatever the life space "is," whatever the psychological facts within it "are," and whatever the regions "are" of which both person and environment consist, certainly one of the most important relations between the parts of the life space is that they exist side by side. The spacial character of these relationships is further strengthened by the fact that we have to deal with a coexisting manifold. The question is whether these are genuine spacial relationships.

We have to meet a great number of objections if we answer this question in the affirmative. These objections have special weight in that they are partly intrinsically, partly historically connected with many philosophical problems. Obviously the quasi-social locomotions, distances, directions, and regions, in the psychological life space are not defined by the space with which physics has to deal. But physical space is the only empirical space about which one is accustomed to speak in a scientific, not a metaphorical way. The final criterion of the applicability of spacial concepts in psychology is their fruitfulness for research. Nevertheless it remains necessary to

[1] That something spacial can appear as the "content" of a perception, of an image, or of a thought is well known, and space perception is one of the most highly developed fields of psychology. It is but a step from this to speak in similar fashion of the perception of a social space, of space of action, or of experiences of spacial structures in general. It is hardly necessary to explain that in this book we will not discuss the experience of space.

clear up these philosophical objections. A few points may be taken up briefly.

Certainly the fact that language employs many metaphors which imply spatial concepts to describe the behavior and fortune of a person (for instance, "He has come down in the world") cannot be taken as proof. Although everyday speech contains important suggestions for the psychologist, such metaphors might rather arouse mistrust against the application of spacial concepts in psychology.

The Mathematical Concept of Space

In order to settle the question of whether the facts that exist psychologically, *i.e.*, those which have psychological effects, show spacial relationships one has to go back to the mathematical concept of space. The person who is not a mathematician as a rule thinks of space as *physical* space. He pictures mathematical space as a geometrical structure extending in every direction without limits, a structure which can be visualized to a certain degree and within which one can determine direction and distance exactly by measurement.

Mathematics, in the course of its long history, has developed a concept of space which is in no way limited to physical space or to space which can be visualized. It has progressed from the investigation of three-dimensional to that of n-dimensional space, from Euclidian space to curved space, from metrical to non-metrical space.

The nature of the things whose system constitutes a mathematical space is entirely irrelevant for modern mathematics. It does not matter whether one thinks of them as physical objects, temperatures, numbers, colors, events, or anything else. Only certain relationships and the possibility of certain operations are relevant. It is these which finally define space (66, pp. 15*ff.*).

As far as mathematics is concerned there is therefore no fundamental objection to applying the mathematical concept of space to psychological facts. The crucial point is whether the relationships that characterize space in mathematics can be applied adequately to psychological facts, and whether one can coordinate psychological processes univocally to mathematical operations.

In order to avoid misunderstanding in terminology it should be noted that one does not think of mathematical space as necessarily unlimited in all directions. One also designates as space a series of numbers, a sphere, or a finite line of any curvature.

Topological Space.—The mathematical concept of space can be developed from different fundamental relationships (31, pp. 211*ff.*). From the point of view of psychology it is especially interesting that one can use the part-whole relationship as the basic one. This basic theorem states that for any two objects, *U* and *V*, of a system for which certain conditions hold, the following relationship shall or shall not be valid: *U* is a part of *V* (that is equivalent to "*V* includes *U*"). By means of certain monotonous series of inclusion one can characterize the concept of a "point"[1] and further the concept of "surrounding."

Space thus defined is called topological space. By this term is meant that we are dealing with mathematical relationships which can be characterized without measurement. No distances are defined in topological space. A drop of water and the earth are, from a topological point of view, fully equivalent. A cube and a sphere also are not distinguishable (27). Nevertheless these nonmetrical spaces exhibit important characteristics which are fundamental also for metrical space. There is a highly developed branch of mathematics which has grown up around the concept of connectedness. It deals with separated and connected spaces, with the different kinds of connectedness, with the relationship of part sets in different regions, with boundaries, with cuts, etc. Problems of dimension can also be treated on the basis of topological concepts without recourse to metrical properties.

[1] The term "point" in these cases is not restricted to point in the usual sense.

It is now generally recognized that the whole-part relationship, and the relationships of the parts to each other play a fundamental role in psychology. This is true for all branches of psychology. The concept of connectedness, for instance the distinction between separate and connected regions, the distinction between different groupings of regions, is as we have shown above of prime importance for characterizing both the person and the psychological environment. Changes of connection, especially the uniting and separating of regions, are just as important for the psychology of perception as for the psychology of intention, satisfaction, or friendship. The basic idea of a person in an environment is in its conceptual content a statement of a certain topological relationship between two regions.

Certain binary relationships, *i.e.*, relationships between two points of topological space (31, p. 210), play an essential role in topology. In this place we must call attention to the topological concept of "path" that connects two points. How parts of a space are connected can be determined to a large extent by the possibility of such paths and the fact that the path does or does not intersect certain boundaries. We will see that the concept of path plays a fundamental role in the constitution of psychological spaces. One can coordinate certain psychological facts which have the function of a psychological connection between two psychological "points" to a "path" which mathematically connects two points. For instance, any kind of locomotion of the person in the quasi-physical, the quasi-social, or the quasi-conceptual field can be designated as a connecting process which corresponds to a topological path.

In addition to the locomotions of the person or other parts of the psychological space we must call attention to that type of real connection which one can call "dynamic communication." The fact that certain regions in the psychological environment and within the person influence other regions, both of the environment and of the person, may be taken as a criterion for connectedness in the topological sense.

In later chapters we will show how psychological and mathematical facts can be coordinated in detail. I hope that it is now clear that from the point of view of mathematics there is no reason why these concepts should not be applied to psychological problems and that psychology has already used, and cannot avoid using, concepts which are of a topological nature. It will be necessary to clarify these concepts and to apply them strictly in order to make them fruitful for the whole field of psychological dynamics.

Metrical Space.—One can ask whether it is possible to go beyond topological determinations and apply metrical concepts to psychological regions. Metrical space is characterized by the fact that one can coordinate to any two points, x, y, of the space a distance $\overline{xy} \geqq 0$ for which certain axioms hold, especially the triangle axiom (31, p. 211). One distinguishes Euclidian and non-Euclidian space according to the particular kind of metrics employed. Common to both is the fact that there are differences in magnitude and that one can use measurement and numbers in characterizing spacial relationships.

Certainly at the present time there are no metrical determinations available concerning psychological life space. It is clear that a double distance in physical space generally does not correspond to a double distance in psychological space. There is no doubt that very different quasi-physical distances often correspond to equal physical distances. A special investigation is necessary to determine whether the triangle axiom whose applicability is the premise for all measurement of distance holds also for quasi-physical fields. It might seem to be impossible to measure in the exact meaning of the word quasi-social or quasi-conceptual distances. The same difficulties appear with the problem of magnitude of layers or regions of the person, or the distance between them.

Not only the concept of distance but also that of direction goes beyond purely topological determinations. Mathematically there is a close relationship between measurement of angles and measurement of distances. As a matter of fact the determination of directions in the psychological life space is as difficult as that of distance.

On the other hand, the history of psychology seems to show that it is impossible to represent psychological causal interconnections without making use of the concept of direction. It is presupposed in the concept of "direction of a movement" as well as in the concepts of "tendency" or "force." The concept of "difference of direction" or of "opposing direction" can hardly be avoided, for instance in the theory of conflict (60, pp. 338–339; 54).

Neither can we ignore the fact that in certain cases one can speak definitely about direction, or at least changes in direction. This is true not only for the quasi-physical, but also for the quasi-social or quasi-conceptual fields. The same is true of the determination of distance. Despite all obvious objections to any exact measurement in the psychological life space it seems to be possible sometimes to make definite statements about distance or at least changes of distance. Such a change of distance takes place, for instance, when one approaches his goal in the quasi-social space (see Example 6, p. 48). One can even compare regions within the life space with each other as to their magnitude, for instance, regions of free movement (see Examples 1 to 4, pp. 42–46).

We shall have to discuss the difficult group of metrical problems after we have dealt with the topological questions in detail. We begin with topological problems because they are the more fundamental both from a mathematical and from a psychological point of view.

The Application of the Concept of Space and Physicalism

The objection has been raised against our representation of the psychological life space by means of topology that it implies an application of physical concepts to psychology.

In another place we shall have to take up briefly the relationship between psychology and physics. Here we want to emphasize only that the application of topological concepts in psychology is entirely independent of this question. The objection is based on a misunderstanding of the position of mathematical concepts in the concept-formation of the single sciences.

The historical development of sciences brought it about that the most broad and many-sided applications of mathematics are in physics. The bond between mathematical and physical thinking has become so close that in many cases one has lost the feeling for the difference between the two kinds of concepts. Nevertheless, there can be no doubt that in physics we are dealing not with pure mathematics but with applied mathematics (7, pp. 34–35). The more recent developments in physics, especially the theory of relativity, have again brought to the fore the importance of this fact.

Not so long ago it was thought that even the application of numbers to psychology was an unjustifiable application of physical concepts. As a matter of fact number is employed not only in physics but also in economics and in history. At present there is no doubt that numbers are mathematical tools which may be applied in psychology like "logical" categories. We do not need to discuss the relationship between logic and mathematics in this place. Just as concepts of logic can be applied in different empirical sciences so can those of mathematics. The application of the same numbers does not imply a carrying over of concepts of economics into history, or of physical concepts into economics.

Like numbers, topological concepts are mathematical, not physical in nature. Wherever one has used them, or will use them in physics or in other sciences, it is a matter of applying them in different and equally justifiable ways. The same is true of metrical spaces, *i.e.*, of the concepts of distance and direction. In these cases also we are dealing with mathematical-logical concepts. And modern mathematics does not cease to insist that these concepts do not imply that the elements of the systems about whose relationships statements are made are physical or are in any way determined in regard to their content. This is true also of the concept of "directed magnitude," *i.e.*, vector, which we will apply in representing certain psychological facts. Like other mathematical concepts the concept of vector can be used to represent facts of very different content. It is not to be confused with the concept of force, certainly not with the concept of force in physics. To use spatial concepts in psychology means to treat psychology mathematically, but does not imply physicalism.

CHAPTER VII

PSYCHOLOGICAL SPACE AND PSYCHOLOGICAL DYNAMICS

Problems of Pure Mathematics and Problems of Coordination

The fact that we have to deal with applied mathematics in the empirical sciences means that the question of which mathematical concepts we have to use for the representation depends in each case upon the characteristics of the special empirical facts. We are not dealing with questions of pure mathematics. Our task is to represent certain empirical data adequately. The mathematical concepts which are used are very different for the treatment of different problems even within one science, and it may be possible that mathematical concepts are adequate for the whole field of one science which cannot be applied at all in another. For instance, it may be that to represent the psychological life space one has to use mathematical space concepts which are entirely different from those now used to represent physical space.

The application of mathematics to psychology involves two kinds of questions. It is important that we be clear in regard to the fundamental difference between them. According to Reichenbach (73, p. 5) we can designate the one kind as problems of coordination, the other we shall call problems of pure mathematics.[1] The problem of coordination is essentially an empirical problem in that its task is to demonstrate the nature of certain empirical facts, in our case facts of psychological dynamics, and to coordinate to them mathematical concepts

[1] Blumberg and Feigl (4*a*, pp. 289*ff*.) survey the philosophical problems connected with the concept of coordination. They contrast purely formal or implicit definitions with applicational (coordinating) definitions.

which represent the logical structure of these empirical relationships adequately. Of course a correct coordination presupposes a sufficient knowledge of these mathematical-logical concepts to which one coordinates the empirical facts. That requires an orientation in regard to the purely mathematical problems involved. In some cases one may be faced with mathematical questions which mathematics itself has not yet treated.

In actual research, problems of coordination and purely mathematical problems are so closely connected that it is not always easy to distinguish between them. Nevertheless, as the history of physics shows, the greatest possible accuracy in this respect is important for the progress of science.

The Instability of Psychological Situations

The instability of psychological situations has often been emphasized. One might easily think that it constitutes a difficulty for the representation of the psychological life space by means of mathematical concepts. It is quite true that the life space or parts of it often suffer strong and sudden changes, but this is equally true of certain physical situations. For the application of mathematics it is quite irrelevant whether a given situation represents a section through an event which occurs with great rapidity or whether one has to deal with a relatively constant situation. Besides, even psychological structures can show a surprising degree of constancy that sometimes continues over a period of years.

Necessary Conditions for the Application of Metrical and Topological Concepts to the Life Space

More essential is a second question which is related to the problem of the indeterminacy of psychological events (see pp. 39*ff*.). As we have mentioned before, one can sometimes determine direction and distances within the life space, that is, one can determine quasi-metrical characteristics which go beyond purely topological ones. In agreement with mathematical usage one could speak here of a "metrisation"

(*Metrisierung*) of psychological life space (86; 16, pp. 14*ff*.), and thereby mean the actual development of psychological organizations which allow of metrical predication.

On the other hand, it is possible that there are cases in which the psychological life space does not even show those characteristics which are a condition for the application of topological concepts. For instance, one could think that the perceptual field of the newborn child is still so undeveloped that it does not yet fully allow the application of the most simple concepts of connectedness or the concept of "parts," and that there only gradually takes place what we could call a "topologisation" of the perceptual field.

A similar development may sometimes be observed with quasi-conceptual fields of the adult, for instance, when he starts to treat a very obscure and complicated problem. It can happen that in a certain part of the life space there occurs a further structurization, or a destructurization. An undifferentiated region, that is, a region in which it is impossible to distinguish certain parts, may become gradually more differentiated, so that one can determine parts and subparts. This makes it possible to make certain topological statements about these subparts. On the other hand if a region is completely undifferentiated the psychological conditions are lacking which would permit topological determinations of its parts.

To what extent topological concepts can be used in representing the life space depends upon the real properties of the case at hand. It seems to me that one of the most important general characteristics of the psychological life space is that it is not infinitely structured, but that it is always structured only to a certain degree. The representation of the life space will have to take this fact into account. It would be incorrect to presuppose in the analysis smaller subparts than actually exist. That the topological treatment of the life space is limited in this way is a fact which is of fundamental importance from a dynamic as well as from a mathematical point of view. For the spaces which are familiar to the mathematician can be subdivided *ad infinitum*. We shall come back to this fact later. For the present discussion of topological questions it suffices to say that the life space is to a certain extent structured. At least there is a certain topological structuring of the environment in nearly all situations with which psychology deals, and no doubt there is always some structuring of the person. This supplies the empirical premise for the application of the topological concepts to the life space.

Space and Dynamics. The History of the Concept of Space in Physics and Psychology

In using mathematical space concepts in psychology one will have to base these concepts on psychological dynamics from the beginning.[1] In this respect the history of psychology is essentially different from that of physics.

In physics there has never been a question as to whether spacial concepts should be used for the representation of structures and events. Physical space is given visually. It was from the beginning an essentially metrical space in which distance and direction were relatively easy to determine. One was able to employ fixed measures which were represented simply enough by solid bodies.

Therefore in the beginning the properties of physical space were not made dependent on dynamics, *i.e.*, not upon the laws of the physical processes. For this reason the problems of measurement in physical space often seemed like problems of pure mathematics. Only the most recent development in physics, especially the theory of relativity, has revealed the direct connection between properties of physical space, especially its metrical properties, and questions of physical dynamics. It has shown the importance of the velocity of light and of other physical facts for measurement. It has further revealed the connection between determinations of time and of space, and it has demonstrated that even if we use solid measuring rods the nature of the physical processes themselves must be taken into account. The determination of physical space therefore, in the last analysis, goes back to physical causal

[1] The same is true of the use of spacial concepts in sociology.

chains (*Genesereihen*). Which particular mathematical space is to be used for the representation of physical structures depends, as modern physics has shown, on the special laws of physical dynamics (73). Nevertheless, physics was able to advance very far in investigating the physical world without having to consider the relationship between the measurement of physical space and the physical laws.

Psychology, in this regard, is in a different situation. The relations of "togetherness" in the psychological life space are given only to a limited extent. As far as the quasi-physical facts of the environment are concerned the spacial relations can to a certain degree often be experienced directly. This is less the case with quasi-social facts and still less so with regions within the person. We do not have to discuss at this time whether this difference between physics and psychology is a fundamental one or only one of degree. In any case psychology, in determining the life space metrically or topologically, is not in possession of tools which in their immediacy and simplicity are comparable to the solid measuring rods of physics. One can represent the order of togetherness of the facts which exist for psychology only if one realizes from the beginning that the determination of spacial relations has to be based upon psychological processes and depends therefore on the nature and the laws of psychological dynamics. For this reason psychology is already being faced with questions which in some respects are as "general" as those which recent years have brought to the fore in regard to physical space.

The Fundamental Concepts of Dynamics

That the spacial structure of the life space depends on psychological dynamics itself shows the importance of the dynamical concepts. These are such concepts as cause of change, tendency, resistance, solidity, equilibrium, force, tension, etc. At present it does not seem to be fruitful to discuss the difficult general problem of which concepts one has to use to represent the dynamics of a certain science or according to what criteria one must select them. I shall limit myself to a few remarks that ought to help forestall the misunderstandings which seem most likely to occur in regard to these questions.

1. If we want to derive actual psychological events conceptually, we have to try to represent not only the spacial relationships but also the dynamic ones in a mathematical way. For instance, we shall often have to substitute the concept of "force," which is clearer and which can be represented mathematically, for the more nebulous concept of tendency.

Nevertheless, it is important to keep in mind that the dynamical concept and its mathematical representation are not identical. The same mathematical concept can be used for the representation of different facts. For instance, a vector can represent not only forces but also movements. It will be necessary to distinguish between the formal, mathematical properties of the dynamic concepts and their content, which is determined by the psychologically real events to which they are coordinated.

2. It must not be assumed here that all empirical sciences should use the same fundamental concepts for the representation of dynamics. Especially it must not be assumed that psychology has to use the same dynamic concepts as physics. Only an investigation of psychological dynamics itself can decide which concepts are adequate for it.

3. As far as we can see at the present time psychology will have to use dynamic concepts that in some respects show a formal similarity to the concepts of physics, but which are not identical with them.

For instance, both sciences can use the concept of "equilibrium." This implies that one also assumes a concept of force, since there is a close connection between the concept of equilibrium and that of force in so far as equilibrium is a special case of a constellation of forces. As a matter of fact, as we shall see, the concept of force and a number of related concepts can be used in psychology in an exact way. Köhler's argumentation (43) has made it sufficiently clear that the same concept of "dynamic whole" or "gestalt" can be used in physics and in psychobiology; and also that the fundamental gestalt laws are equally valid for both sciences.

When we use equivalent concepts of this sort we are not trying to derive psychological from physical concepts. The content of these dynamic concepts is determined only by coordinating them with real psychological or real physical processes. In so far as one coordinates these concepts to different real processes, in the one case to psychological processes, in the other to physical ones, these conceptual fields remain separate from the point of view of theory of science. When for instance, in the following we speak of forces in connection with the quasi-social or quasi-conceptual facts in the life space we define the forces as psychobiological and not as physical. The application of formally equal dynamical concepts in psychology and physics does not mean a regression to physics any more than the application of the same mathematical concepts. In both cases we use processes which are different in content for the definition of dynamical concepts which are equivalent in form.

Besides, this formal correspondence between the dynamical concepts of physics and those of psychology seems to hold only within limits. For instance the laws which pertain to forces are probably essentially different in form for the two sciences.

4. In spite of this lack of complete correspondence we have often chosen terms that are also used in physics to designate our psychological dynamical concepts. The use of identical terms for the two concepts rather points to their functional equivalence within the system of dynamic concepts in physics and psychology, than to a complete identity in a formal mathematical sense. Yet at the same time I shall try to avoid using identical terms except when the concepts are at least similar in form. It would have been easier to decide whether to introduce a completely new terminology into psychology or whether to make use of existing terms if we could have foreseen exactly how far the formal similarity between the concepts would hold. But this we can learn only by an investigation of psychological dynamics itself. Actually there is probably not much danger of misunderstanding as long as one holds to the definitions as we shall give them.

CHAPTER VIII

THE PSYCHOLOGICAL WORLDS AND THE PHYSICAL WORLD

If one is accustomed to speak of psychological facts as something essentially nonspacial, one thinks first not of mathematical but of physical space, *i.e.*, of the togetherness of the facts that are real in the sense of physics. An essential characteristic of this physical space is that it is thought of as a single coherent space which includes the totality of all physical facts that exist at a certain time[1] (the whole physical world) and which includes only physical facts. The facts of psychology, *i.e.*, these facts which psychology must recognize as real, have, according to the teaching of physics, no place within physical space. This is equally true of economic or aesthetic facts.

Physical and Psychological Space

The thesis that psychological facts, such as dreams and wishes, have no place in physical space has long been accepted as self-evident by the physicist. This thesis is also generally accepted in psychology and constitutes the main reason for the assumption that psychological facts are in general of a nonspacial nature. Nevertheless attempts are continually being made in psychology to "coordinate" dynamic facts in psychology to physical facts.

It would lead too far to enter into the tangle of single heterogeneous tendencies of different value that are presented by the followers and the opponents of this procedure. It seems to me that one can solve this complex of related problems only if one attacks it from the point of view of the theory of science. This means that this problem must not be separated from the more

[1] We can disregard the complications that the theory of relativity introduces into this concept.

general problem of the relation of the different sciences to each other, *i.e.*, that instead of contrasting "physical" and "mental" one has to think in terms of the difference between physics and psychology, or of the difference between physics and biology (53).

Then two oppositions result: (*a*) A distinction within biology between bodily and mental; (*b*) a much more fundamental distinction between "object of physics" and "object of biology" (including psychology). The main reason for many confusions is that the term "physiological" is used in two different ways; sometimes it is used to designate a certain class of biological events, *viz.*, "bodily" processes in opposition to psychological ones and sometimes to designate "object of physics." The difference between the complex of "physical" questions on the one hand and "psychological-physiological" problems on the other is based on the difference in the way the temporally distinguished units of the existential series (*Genesereihen*) (53) are related to each other. From this difference in the temporal order follows the difference in the spacial order in physics and psychology.

We must emphasize that not only psychological facts but also the "bodily"-biological facts do not belong to the physical space. This makes it clear that in the following when we speak of psychological regions, forces, or changes, we are not dealing with figments of the imagination but with facts which have the same reality and kind of existence as biological facts in general.

The Singleness of the Physical World and the Plurality of the Psychological Worlds

Despite the changes in the concept of physical space during the recent development of physics one has always considered physical space as one single connected space that includes all existing physical realities. Psychology, on the other hand, does not deal with one single connected space of psychologically real facts. Especially when one takes "effects" as the basis for the construction of psychological spaces, one will have to think of each individual's life space as an entirely separate world. A wish which plays an essential part in the life space of the individual *A* may possess no psychological reality for the individual *B*.

From the point of view of dynamics the life space of each single individual is a totality which is equivalent to the totality of the whole physical world.

Naturally, not only another individual *B* but also thoughts of other individuals can have effects in the life space of a person *A*. But also in this respect the principle which we discussed above in regard to quasi-physical, quasi-social, and quasi-conceptual facts is valid, namely, that one has to take into account only those processes which exist psychologically for the person *A*.

The older psychology of expression upheld the principle that the mental processes of another person could only be inferred. This principle as such is certainly untenable. It is probably based in part on a misapplication of the principle which in itself is correct, that from the point of view of psychology the life spaces of different persons are to be treated as separate worlds which can only affect each other indirectly. (See pp. 69*ff.*)

In this respect there is a sharp distinction between the concepts of sociology and those of psychology. Sociology is not obliged to consider only facts which belong to the life space of one single individual as being in direct dynamic communication with each other. For instance, the dynamics of a group of persons can represent a single system for sociology.

In any case psychology does not deal with one single connected space which represents the totality of its world, as does physics. Instead its subject matter is a plurality of separate spaces, *i.e.*, a great number of totalities each of which corresponds to a single man or animal.

The Physical World as a Dynamically Closed[1] Unity and the Psychological Worlds as Dynamically Unclosed Unities

Physics treats its space as a closed system of causes and effects in the following sense: All physical changes are the result of con-

[1] The term "dynamically closed" should not be identified with the topological term "closed." It is irrelevant for our problem whether physical space is "closed" in the topological sense of the word (see p. 89).

ditions or changes within the same physical space. According to physics there are no influences on this space from outside.

In psychology also there is a more or less close dynamic connection between all the facts which belong to the same psychological space. The psychological events are determined by the life space according to the formula $B = f(S)$. So far therefore, as in physics, a change is the result of conditions or events within the same space.

Nevertheless, the matter is not so simple in psychology. That becomes clear if one asks, in connection with the question of "historical causality" (see pp. 30*ff*), how the situation S came into being. Physics answers this question by proceeding from a temporally later situation, S_2, to a preceding situation, S_1, and shows how S_2 came into being through events which resulted from situation S_1. It can be impossible to derive situation S_2 from the situation S_1 because the situation S_1 which one chose was not sufficiently comprehensive and therefore certain facts were neglected which in the interval influenced the system from the outside. However, theoretically, it is always possible in physics to choose an S_1 so comprehensive that S_2 can be derived from it. The facts of S_1 always exist physically and thereby have a place in physical space.[1] In this sense there is, in physics, no effect from outside on physical space.

In psychology one tries in similar fashion, to derive situation S_2 from a preceding situation S_1. As a matter of fact that is sometimes possible. A person A may, in a situation S_1, have intended to perform a certain action. The event B_1 which results according to psychological laws from the intention and the other characteristics of S_1 may make it possible to determine sufficiently the later situation S_2. That means that in this case we have derived the condition of a life space from that of the same life space at an earlier moment of time. In so far the analogy to the physical world is complete.

[1] We are here dealing with the fact that the physical causal series (*Genesereihen*) can always be traced back in time. That means that there is always an object O_{t-n} which from the point of view of physics is completely "gen-identical" with the object O_t. *Cf.* K. Lewin, (53, p. 49).

But it can also happen that during the event B_1, which results from situation S_1 according to psychological laws, certain events enter which themselves cannot be deduced from situation S_1. Thus, while a person A is busy writing a letter the door may open and an outside person enter unexpectedly. Or, while someone is marking off the place where his house is to be built in a field, a herd of cattle may pass and destroy the lines. Or, when a man is building a fence, there is a slide and the situation is changed entirely. The influence of such events on the behavior of A may again be determined by psychological laws. Nevertheless, in these cases we are dealing with an influence from outside in so far as at least part of the events which lead up to situation S_2 cannot be derived from the situation S_1. Such a derivation would not be possible even if all the psychobiological laws were known.

Here we are faced with one of the principal differences between physical and psychological space. In physics it is theoretically always possible to make S_1 more comprehensive when it does not suffice for the derivation of S_2. In psychology such an expansion is often not possible. If one tries to make a corresponding enlargement and for instance includes the approach of the outside person to the door of the letter writer A in A's life space at the time 1, one makes a mistake. For at that time the stranger clearly does not belong to the life space of A because the behavior of A would have been different if he had known of the approach of the stranger. We cannot avoid recognizing that there are such influences from outside on the psychological life space. That means that there are changes which cannot be derived from the dynamics of the psychological life space even if one assumes strict determinism in psychology and if one has a complete knowledge of the previous situation and of all psychological laws. These changes can only be thought of as influences on the psychological life space which are "alien to psychology."

I do not use the expression "alien to psychology" to indicate physiological or other bodily influences which, as we have seen, have to be included in the system of psychobiological laws. Instead I use it to designate such influences on a situation as cannot be derived from the psychobiological properties of the preceding situation.

Such alien influences occur frequently. The field of perception and action can be changed for instance by the fact that an object is suddenly set in motion by physical causes, that another person encroaches, that the telephone rings, etc. These influences from outside can have a definitely social character. The announcement of new regulations for peddlers, about unemployment relief, about taxes, can entirely change the field of action of a peddler, of an unemployed person, or of the tax-payer. As mentioned above such influences can occur by way of perception or as gross somatic influences. In all these cases we find essential dynamic changes of the life space of an individual which do not depend on the psychological dynamics of the life space itself. The single psychobiological worlds do not therefore represent dynamically closed regions in the sense indicated above.

If one follows up this problem one sees that almost all processes which are based primarily on psychobiological dynamics depend to a certain degree on alien factors. If someone saws a board his behavior is determined not only by his goal but also by the nature of the wood and the properties of his saw. The same is true when a year-old child puts one block on another and finds that they do not stick together however hard he presses them; or when someone tries to influence a political group, or to solve a mathematical problem, and finds that things do not go as he had expected. If someone throws a ball at a mark, if he tries to influence another person through praise or blame, if he goes along a street, or looks around, in all these cases, the actual effects of the psychological event depend also on facts which are alien to psychology.

We have, further, to call attention to the connection between cognition and these alien factors. Cognition has always been treated as a specific characteristic of mental life although it has not at all such outstanding importance for inner-psychological dynamics. This may be one consequence of the fact that the psychological worlds are not closed. Perception and cognition often affect the life space in such a way that the structure of certain of its parts corresponds in a high degree to the objective structure of what is perceived (44). How far and at what points the two structures correspond in concrete cases is very important for the success of an action, and is decisive for the value of a plan. The comprehension of the intrinsic nature of the alien factors, whether of physical relationships, of mathematical problems, or of social groups, is therefore of the greatest importance for every achievement.

It is not always easy to decide, in particular cases, what is to be taken as alien and what as real psychological influences. A child may want to get a cloth to clean up water that the "naughty" doll has spilled. If he asks an adult where to find the cloth, then the adult's answer represents an alien factor, *i.e.*, it cannot be derived from the preceding situation in the life space of the child. But if the child already knows where the cloth is, one is inclined to speak of a simple psychological relationship. Further, certainly the kind and order of visual impressions which the child receives as he walks through the rooms depend upon the arrangement of physical objects in the rooms, yet when the child is thoroughly familiar with his environment one does not think of this fact as constituting a break in the psychological causal chain. For then the change in the visual impressions is a result of the behavior of a child. Nevertheless, even in this case the change in the situation is partly determined by alien factors.

The concept of a world which is dynamically not "closed" but within which there exists nevertheless a strict determinism offers some difficulty at first, and one might ask whether it is at all possible to give a conceptually clear definition of such a space. Closer examination, however, shows that a mathematical representation of such a world is quite possible.

We have to consider two possibilities: The influence "from outside" can affect the psychological life space at every point or only in certain regions.

The mathematical representation of the second case is simple. It means that one has to distinguish within the life space "inner points" and "boundary points." The life space therefore would be a "limited" and "closed" region, *i.e.*, a region which includes its own boundary. (These mathematical concepts will be explained in Chap. X.) The boundary points would correspond to those zones of the life space that can be influenced from the outside. This representation would be correct if for instance all influences on the life space from the outside were mediated by the surface of the body of the person. The boundary of the life space would be a simply connected region which, represented two-dimensionally, would correspond to Fig. 5. Only psychological laws would then govern the inner part of the life space. Alien influences would affect only these boundary points.

The other possibility, *viz.*, that each point of the life space can be affected by alien influences, can also be represented

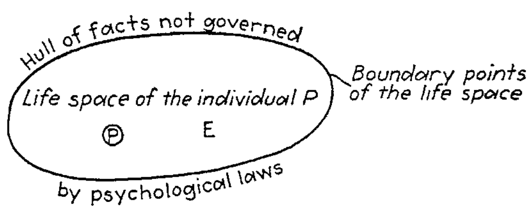


FIG. 5.—The life space as a "dynamically not closed" world. *P*, person; *E*, environment.

mathematically. It would mean that all points of the life space are boundary points. This postulate is fulfilled if one imbeds the space of psychological facts in a space which has one dimension more. Then, regardless of the number of dimensions of the psychological life space, every point becomes a boundary point in relation to the space of more dimensions.[1] Let us assume that the life space could be represented by a two-dimensional manifold, for instance by a plane. Then the events that occurred could be derived according to psychological laws from the structure and the dynamic properties of the facts represented in this plane. Nevertheless, each point of the plane would be a boundary point in regard to a three-dimensional space. This three-dimensional "hull" would make it possible to represent those physical, social, or other facts which are alien to psychology and which do not influence the

[1] To illustrate: Each inner point of a disk is a boundary point in relation to a three-dimensional space in which the disk lies; each inner point of a line, that is of a one-dimensional space, is a boundary point in relation to the plane.

life space at the present moment but which can affect it in the future. Within this hull would hold not psychological, but other laws.[1] In this case the life space could again be "limited" (as in Fig. 5). But it could also correspond to the unlimited plane. An analogous possibility of a transition to a hull of more dimensions exists mathematically also in the case in which the life space itself represents a manifold of three, four, or more dimensions.[2]

In answering the question how the boundary points in the life space are distributed one will have to take into account that not only is the life space influenced from the outside, but that effects can also work in the opposite direction; that is, behavior can affect those regions which are not subject to psychological laws.

One must not conclude from the unclosedness of the psychological worlds that there is no use in speaking of a psychological or a psychobiological causal relationship. The task of dynamical psychology is to derive univocally the behavior of a given individual from the totality of the psychobiological facts that exist in the life space at a given moment ($B = f(S)$). To this also belong all those facts at the boundary points which influence the person at the present moment but which themselves owe their existence partly to alien events. In so far, therefore, the task of deriving the behavior B from the totality of S remains unchanged and has not lost anything of its psychological character. In so far there is no formal difference between psychology and physics. The difference consists in the fact that there are no boundary points in the physical world which depend on nonphysical factors.[3]

[1] In constructing this non-psychological hull one would probably therefore have to go backward in time. In that respect this hull would be formally similar to other fundamental "*genese*" relationships in biology, for instance pedigree. *Cf.* K. Lewin (53, pp. 83–85, 144).

[2] In mathematics, too, one distinguishes the questions of the inner structure of an n-dimensional space from its relation to its n-plus-one-dimensional hull. *Cf.* K. Menger (66).

[3] It is irrelevant for this discussion whether this conception of the physical world is correct.

It is obvious that psychology must take into account also physical and social facts which obey non-psychological laws and which control the events in the "foreign" hull of the life space. For these facts determine the boundary points of the life space and are therefore of great importance for all events in it. Every act of influencing another person, whether in laboratory experiment or in everyday life, consists in creating such a hull, one which affects the boundary points of the life space and thereby the life space itself in a certain way.

To summarize: psychology has to assign a separate space to each single individual and his own environment. Each such space corresponds to the totality of a psychobiological world. (From the point of view of theory of science it is equivalent to the whole physical world.) These worlds are "dynamically not closed"; they have boundaries or each of their points exhibits boundary properties in relation to certain influences which are alien to psychology.

CHAPTER IX

MATHEMATICAL REPRESENTATION AND PSYCHOLOGICAL THEORY

In concluding this general part I want to discuss shortly several points of our procedure which have been misinterpreted.

CONCEPT, PICTURE, AND SYMBOL

Our aim in representing a psychological situation is not to give a visual picture of it. Illustrations may serve as a pleasant pedagogical device, but this is not of essential importance for our real task. In recent times one occasionally finds diagrams used in different ways in psychology.

I select two examples: Charlotte Bühler (12, p. 237) illustrates the change in the relation of familiarity to like and dislike with the diagram which is given in Fig. 6. It is char-

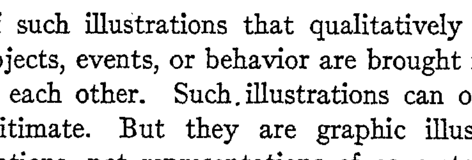


FIG. 6.—[*From Ch. Bühler* (12).]

acteristic of such illustrations that qualitatively determined classes of objects, events, or behavior are brought into certain relations to each other. Such illustrations can of course be entirely legitimate. But they are graphic illustrations of abstract relations, not representations of concrete situations. Further examples of geometrical concepts in psychology that do not represent spacial relations of the life space are the concept of dimension as used by Boring (5) in referring to dimensions of properties, the concept of φ-manifold as employed by N. Raschevsky (72) in discussing the biophysics of space and time, and the concept of vector as used by Thurstone (83) in connection with factor analysis.

More closely related to ours are representations of the kind which Koffka uses to make clear the relation between behavior and environment. "*G* is the geographical environment. It produces *BE*, the behavioral environment; in this and regulated by it *RB*, real behavior takes place, and some parts of it are revealed in *PHB*, phenomenal behavior. In some sense *BE*, *RB* and *PHB* occur within the real organism, *RO*, but not in the phenomenal *Ego*, which belongs to *PHB*. *RO* is directly affected by *G* and acts back upon it through *RB*" (47, p. 40) (see Fig. 7). This illustration is more nearly concerned with the task of representing the structure of the life space than that of the previous example. But it is clear that this figure

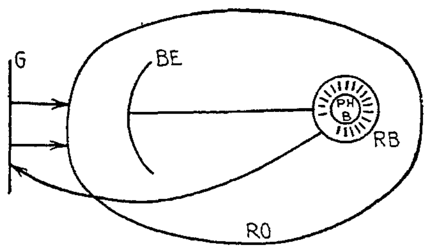


FIG. 7.—[*From Koffka* (47).]

is meant only as a "picture" of the structure and not as a mathematical representation in our sense. If this were not so it would be incorrect to say of the behavioral environment (*BE*): "In this . . . *RB*, real behavior takes place" and nevertheless to represent in the figure the behavioral environment as a one-dimensional limited line which does not enclose *RB*. One would have to ask further why behavior, *RB*, is represented as a two-dimensional region while the behavioral environment *BE*, and the geographical environment *G*, is represented as a one-dimensional line.

In the following we are not dealing with the representation of the dependency of certain classes of events on each other nor with pedagogical aids to visualization (28, p. 170; 49, p. 161) but with the conceptual determination of the dynamic properties of concrete situations. That we generally illustrate this conceptual representation by a figure is a matter of secondary importance. We even have to emphasize that the figures lead to misunderstanding if a reader interprets them in terms of the usual metrical geometry instead of topology. The diagram on the paper is in fact only a picture of certain topological structures which for their part are to serve as a conceptual representation of psychological facts.

This means that it is not our aim to find arbitrary symbols for the representation of situations. The mathematical concepts should "picture" the dynamic properties of the situation only in the sense in which concepts represent facts.

One can distinguish in our figures between the arbitrary symbols (for instance, that the point of application of a force is indicated by the head of an arrow; that the strength of a force is indicated by the length of the arrow; that the strength of a barrier is shown by the thickness of a line) and the representation of the topological relationships themselves (for instance, the closedness or not-closedness of a curve).

If one wants to take the term symbol in a broader sense, one can even consider concepts as symbols for real events. But that would mean that the use of mathematics in physics is of a symbolic nature.

The mathematical concepts which we shall use in the following for the conceptual representation of psychologically real facts are to be understood no less strictly than mathematical representation in physics. Mathematical concepts are distinguished from other means of representation, such as the symbols of ordinary speech, in that they belong to a system of concepts which are related to each other in a univocal way. The scope and unambiguousness of these relationships are what makes the coordination of mathematical systems to real facts so fruitful for investigation and this is no less true for psychology than for other sciences.

Concept and Model

We have intentionally avoided the use of any model of a physical or of a nonphysical nature for the explanation of psychological dynamics.

Like an illustration the working out of a model can have a certain value. On the other hand it can, especially in psychology, involve serious dangers: a model usually contains much that is purely arbitrary. One usually uses it like an illustration only in so far as the analogy holds, *i.e.*, really only as long as it is convenient. As soon as consequences ensue which do not agree with the real facts, one evades the difficulty by asserting that it is after all only a model or an illustration. One says, "A comparison is not an equation." How far one uses the model for explanation and at what point one discards it as no longer binding is purely arbitrary. In this respect model (7, p. 53) and illustration are sharply distinguished from the mathematical representation which we are trying to attain. If one decides to represent a real fact by a mathematical concept then one is forced to acknowledge all the consequences which are involved in this concept. This certainly makes the task a difficult one. On the other hand science will obtain the real benefit of the application of mathematical concepts only if it uses them in an absolutely binding way.

Physiological Theories of Psychological Processes

One must welcome every attempt to go beyond vague ideas to concrete formulations in psychological dynamics. Often psychologists who take this task seriously have been driven to attempt to make "physiological" theories. I do not consider that tendency fortunate. Certainly one cannot object in the least to applying physiological methods and to including in the theory material so obtained. This is in line with the general standpoint of this book and does not need to be especially emphasized. But the so-called physiological theories which are based on psychological facts have almost always the character of a physiological or rather physical model of which the same is to be said as of other models. With a physiological model too, the relationships with which one is concerned are not expressed directly in concepts but only indirectly in illustrations, and these often include superfluous specializations. The task of scientific research is, however, to determine the dynamic characteristics of the facts themselves.

At present, there can be no more doubt that such a determination of the dynamic properties of the underlying facts is also possible by "psychological" technique in the narrower sense, for instance by psychological experiment. We have to determine more and more exactly the properties of these dynamic systems which have full psychobiological reality. It is therefore meaningless to duplicate these dynamic systems by coordinating physiological systems to them, since the properties of the dynamic systems which are known by means of psychological technique are already the properties of the real psychobiological systems themselves. It would not be correct to apply the concept of isomorphism, which has genuine value in other connections (45, p. 38), to the relationship between "psychological" and "physiological" systems. For at the level of dynamics we are not dealing with a duality but with identical systems.

Working out a problem in terms of a physiological model can sometimes have heuristic value. Very often however it only creates the illusion of a transition to a greater degree of reality without advancing the conceptual determination of the dynamic properties. When Köhler wants to characterize dynamic properties of psychological facts he often does it by speaking of the properties of the "brain field." For the reasons given above we do not follow him in this terminology[1]; but at the same time we want to call attention to the fact that the structure of the brain field coincides with the structure of what

[1] As a matter of fact it seems to be difficult with such a terminology to avoid the danger of referring too directly to the most simple fields of physical forces. Köhler says, "As the distance is enlarged objectively, exactly the same occurs in the brain field . . ." and " . . . physiologically, the increase of distance in the brain field will correspond exactly to the stress which, as a field of force, was tending in that same direction" (44, p. 390). To conclude from these propositions that the strength of a psychological tension is a monotonous function of the distance between a person and his goal would not agree with the results of experimental investigations. Besides, the relation between the magnitude of the tension and the magnitude of the forces, under the conditions in question, is in physics also not a simple one.

we call life space, certainly in its main features. Köhler distinguishes within the brain field regions which correspond to the surrounding field and a special region which corresponds to the person himself. He coordinates to locomotions of the person changes of the position of this region in the brain field. Indeed the methods which Köhler uses in determining the structure and the properties of the brain field agree essentially with the methods which serve us in determining the structure and properties of the life space. The agreement is shown especially in that questions of dynamical connection and of relations of position play an especially fundamental role. If one tries to make an explanation in terms of dynamic relations, one comes to assumptions about the structure and the properties of the basic facts that seem to be equivalent, at least in their mathematical form, regardless of the terminology in which they are expressed. One must always go back to these invariant relations in the last analysis if one wants to derive the behavior of a person conceptually. It seems to us therefore that the principal task of any dynamic psychology is to investigate and represent directly these relations.

Representation and Explanation

The objection has often been raised against our representations of situations that it is self-evident that one can derive from the represented situations the events one wants to explain. It is said that our representations do not explain, that they only describe.

If one wants to understand the meaning of this objection, one must remember that one has often explained psychological processes by deriving them from entities "behind" the processes. (The concept of instinct has often been used in such a way.) The derivation of events which we give here implies only one kind of tracing back, namely, the progress from phenomenal to conditional-genetic characteristics of the objects and events themselves (60, pp. 318–321). This however seems to me to be not a defect but rather one of the most important positive characteristics of the new way of building theories.

The task of dynamic psychology is to find the psychological laws and to represent the situation in such a way that the actual events can be derived from it in a conceptually univocal manner. If the objection is raised that it is self-evident that the events follow from these representations of situations, one has to answer that this is exactly our purpose. What we are trying to do is to represent situations in such a way that the events follow from them "self-evidently," namely as purely logical consequences.

If one wants to call this "description," it is not worth while to quibble over words. But if one considers conceptual derivation and the transition from phenomenal to dynamic facts as the characteristics of an explanation, then what we have here is in fact explanation. And we would like to add that this is the only kind of explanation which psychology can acknowledge after the transition from Aristotelian to Galilean ways of thinking.

These somewhat subjective objections to the new way of thinking show again how similar, from the point of view of theory of science, the present state of psychology is to the state of physics at the time of Galileo and Newton. In that period of physics one can observe a very similar change in the meaning of explanation, namely, a repudiation of theories of the older style which do not seek an explanation in the relationship of the dynamic facts themselves but in entities which are "behind these facts" (15; 60).

We hardly need mention that theories and working hypotheses are not lacking in the new procedure. The dynamic structure of a situation is not an immediately given fact. As we have said the complete representation of even one given situation would presuppose the solution of all psychological problems and the knowledge of all psychological laws. For scientific research the difficulties begin as soon as one tries to represent a "given" situation. A complete representation of one situation would mean that the whole task of psychology is completed. The representation can be made only step by step and its progress must be parallel to the investigation of the dynamic laws.

The representation of a situation implies no less theory than the laws which it presupposes. It is important that the investigator be fully aware of this. At the new stage of development the representation of facts is so closely interwoven with the formulation of laws that one can say, in regard to dynamic facts, "A correct representation of what 'is' is at the same time an explanation of what happens."

We do not want, in this place, to continue the discussion of methodological questions nor to treat further the properties of the life space as a whole. For one comes to the answers to these fundamental questions only as one progresses with the special problems of psychology. These special problems and the value and usefulness of mathematical means of representation are to a certain extent independent of the way in which one solves the general problems.

PART TWO

TOPOLOGICAL PSYCHOLOGY

A. CONCEPTS OF TOPOLOGY WHICH ARE FUNDAMENTAL FOR PSYCHOLOGY

There are two groups of concepts which are fundamental for the representation of psychological situations. They are intimately connected and make up the framework of the whole system.

1. Formal mathematical concepts (for example boundary, region, connection, vector).

2. Psychological-content concepts (for example solidity of boundaries, fluidity of materials, strength of psychological forces).

The formal mathematical concepts shall serve as guides in our presentation. To be sure, one can apply them in psychology only if one coordinates them correctly to psychological contents that are finally defined by observable processes.

Since the necessary mathematical sources are not always easily available for the psychologist, it might be worth while to begin with a short survey of these concepts. We have tried to present these mathematical concepts in a simplified form which is adapted to the needs of psychology.[1]

According to a mathematical point of view, I have arranged our questions in two groups:

a. Topological problems.

b. Vector problems.

In general one may say that the topological tools allow us to determine which events are possible in a given life space and which are not possible. Vector concepts are necessary to determine further which of the possible events will actually occur in a given case. Accordingly within general problems of psychological dynamics we can roughly distinguish between

[1] It is not our task to derive these concepts mathematically from each other. We only want to make them intelligible to psychologists.

problems of *topological psychology* (presented as Part Two of this book) and of *vector psychology.*

One may further enumerate the following groups:

c. Problems of dimension. (These problems will be treated with the topological problems.)

d. Inducing fields.

e. Tensions. (Topics *d* and *e* will be treated in connection with vector psychology.)

CHAPTER X

CONCEPTS OF TOPOLOGY FUNDAMENTAL FOR PSYCHOLOGY

The determination of topological relationships is the fundamental task in all psychological problems. Changes of connection are the most important changes both in the psychological environment and in the structure of the person. At the same time the topological relationships are fundamental for the mathematical side of our problem.

Topology, as the most general science of spatial relations, can be based on the relationship between "part" and "whole" or in other words on the concepts of "being-included-in."[1] Closely related to these concepts is that of the "surrounding" of a "point."

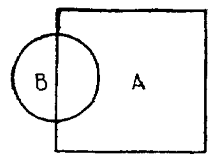


FIG. 8.—Topological sum and intersection of regions.

If A is a part of B one can write $A < B$ (A is a part of B) and $B > A$ (B includes A). By $A + B$, the "sum" of A and B, one understands the totality of all points which are included in either A or B. By the "intersection" of A and B (the part which is common to A and B) is meant the totality of points which are part of both A and B. One expresses intersection as $A \cdot B$. In Fig. 8 the sum of the rectangle A and the disk B is a rectangle with a half disk. The intersection is the half disk which lies within the rectangle.

If A is part of B then $A + B = B$ and $A \cdot B = A$. Further, for every part of a space the equation $A + A = A = A \cdot A$ holds. Two regions A and B are called "foreign" if they have no common parts, *i.e.*, if the intersection $A \cdot B$ equals 0.

[1] For this and the following definitions *cf.* K. Menger (66, p. 17); F. Hausdorff (31, p. 244); W. Sierpiński (79); O. Veblen (87).

The Concept of Region. The Connected Region

We shall start with the topological concept of "region." A region is called "connected" if every point of it can be connected with every other point of it by a path which lies entirely within the region. The region represented in Fig. 9 is therefore connected. The region in Fig. 10 consisting of *B* and *C* is not a connected one. The region consisting of the points in Fig. 11 is also not a connected one. Thus the concept of region does not imply that its parts must be connected.

The region *B* in Fig. 10 is in itself a connected one just as the region *C* is. From a topological point of view there is no difference between the regions *A*, *B*, and *C*. One can use as

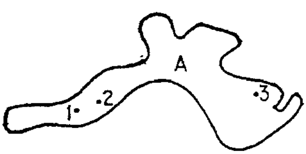


Fig. 9.—Connected region.

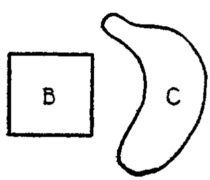


Fig. 10.—*B* + *C* is a not connected region.

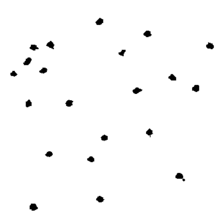


Fig. 11.—Not connected region.

criterion of the topological equivalence the following fact: by a process of continuous transformation it is possible to convert any one of these regions into any other without changing the connections within the region, *i.e.*, by stretching or bending without tearing. Topologically there is no difference between a circle, an ellipse, a regular or irregular polygon with any number of sides, and the figure *A* (*cf.* Fig. 9). Likewise, there is no difference between a sphere, a cube, a cylinder, and a cone.

Differences in size are also disregarded in topology. There is no topological difference between a drop of water and a sphere the size of the sun. One cannot say that the distance between points 1 and 2 in Fig. 9 is less than the distance between points 1 and 3. The impossibility of determining size refers not only to distances but also to angles. Nevertheless, a number of important mathematical statements can be made about spacial relationships of such topologically defined entities.

Closed and Open Regions

Topology distinguishes between "open" and "closed" regions. Examples of closed regions of two dimensions are a disk with its boundary, or the regions *A*, *B*, and *C* in Figs. 9 and 10, if one includes the contour as part of the region. Examples of open regions are the inner region of a disk, the regions *A*, *B*, and *C* without their boundaries, or an unlimited plane.

An open region is usually characterized as a region for each point of which there is a surrounding that lies entirely within the region.

For a closed region this characteristic does not hold: each surrounding of a boundary point contains points which do not belong to the region. This is true however small a surrounding one may choose for the boundary point. A closed region is therefore characterized as a region which includes its boundary points.

Limited and Unlimited Regions

The distinction between "open" and "closed" regions is not to be confused with that between "limited" and "unlimited" regions. An example of an unlimited two-dimensional region is a plane; of a limited two-dimensional region a triangle. The limited regions within a plane can be visualized by the fact that one can draw a closed curve in which they lie. An open region can nevertheless be limited (examples: the inner regions of the Figs. *A*, *B*, and *C*). The plane is an open, unlimited region.

Simply and Multiply Connected Regions

The connected regions *A*, *B*, and *C* in Figs. 9 and 10 and likewise region *D* in Fig. 12 are simply connected. In order to understand what this means we must first consider another concept. A path which connects two boundary points of a region and which, aside from these boundary points, lies wholly in the inner part of the region is called a "cut." A simply connected region is defined by the fact that any cut destroys its connectedness. For instance, the cut c divides the connected closed region D of Fig. 12 into two parts D_1 and D_2 such that it is impossible to connect a point of D_1 with a point of D_2 by a path without crossing c or some other part of the boundary of D_1 or D_2.

A cut does not necessarily have this effect in every connected region. For instance, the cut c in region E of Fig. 12 does not destroy its connectedness. This region has the character of a ring and its boundary consists of two closed curves m and n. In spite of the cut c every point of the region E can be connected with every other point by a path which lies wholly within E. But if one adds to the first cut a second one, E is

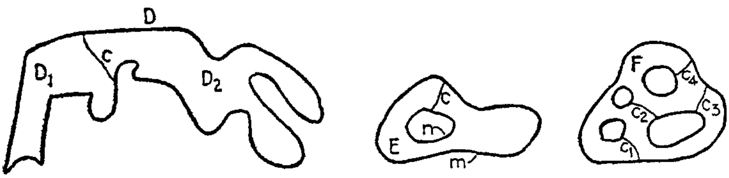


FIG. 12.—D is a simply connected region. E is twofold connected. F is fivefold connected.

no longer connected. Such a region is therefore called "twofold connected."

Region F in Fig. 12 is "fivefold connected." It is possible to make four but no more than four cuts in such a way that F remains connected, for instance, the cuts c_1, c_2, c_3, and c_4.

The reader may get an idea of the binding nature of these topological properties by trying out the different series of cuts that can be made in F without destroying its connectedness.

Jordan Curve, Boundary, Path

It is necessary to discuss briefly the mathematical characteristics of the simply connected closed regions. Such regions, in a certain sense, represent the simplest case for dynamic problems in psychology. We shall use them as our most important elements of construction both for the problems of the psychological environment and for those of the person.

The boundary of a simply connected limited region has the character of a Jordan curve. A Jordan curve is defined as a topological, *i.e.*, as a one to one continuous, image of a circle. It is therefore a closed curve which does not intersect itself. Aside from this the shape of the curve is irrelevant. The borders of the regions *A* (Fig. 9), *B*, *C* (Fig. 10), *D* (Fig. 12) are Jordan curves. On the other hand, the boundary of the region *E*, which consists of two separate curves (*m* and *n*), is not a Jordan curve. Likewise the boundaries of region *F* (Fig. 12) and of Fig. 13 are not Jordan curves.

FIG. 13.

The Jordan curve has several characteristics which make it of fundamental importance for our representations. It divides the plane into two regions, an inner and an outer one, in the following sense (*cf.* Fig. 14). Each point of the outer region *O*, for instance 1, can be connected with any other point of this region, for instance 2, by a path (*m*) which does not intersect the Jordan curve. Likewise one can connect every point of the inner region *I*, for instance 3, with every other point of the inner region, for instance 4, by a path (*n*) which does not intersect the Jordan curve. On the other hand each path (*r*) which connects a point (5) of the inner region with a point (6) of the outer region intersects the Jordan curve. The Jordan curve is in this well-defined sense the boundary between the inner and the outer regions.

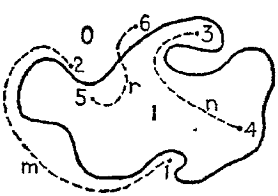


FIG. 14.—Jordan curve determining an inner region (*I*) and an outer region (*O*).

By path one understands the connection between two points by a Jordan arc, *i.e.*, by a part of a Jordan curve. A path is therefore a curve which does not intersect itself.

FOREIGN REGIONS

Some characteristics of boundary and connectedness which are essential for psychology become still clearer if one starts from the concept of "being foreign." As we have explained, two regions A and B are called foreign if they have no common part, or more correctly expressed, if the intersection of A and B is empty ($A \cdot B = 0$). It is important for psychology that it is possible to distinguish between different cases in this respect.

The open regions B and C of Fig. 10 are foreign and likewise the open regions D and E of Fig. 15. But while the boundaries (b) of B and C are foreign ($b_B \cdot b_C = 0$), there are boundary points of D which at the same time are boundary points of E, that is, the boundaries of D and E have an intersection which is not empty ($b_D \cdot b_E \neq 0$). Any boundary serves both to separate and connect two regions. This double function is important for psychology.

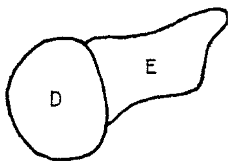


FIG. 15.—Foreign regions with not foreign boundaries.

The difference between the relation of the regions in Fig. 10 and those in Fig. 15 can further be expressed as follows: The sum $B + C$ of the closed regions B and C is not connected. The sum of the closed regions D and E is connected. We can now give one more definition of the concept "connected": A region (which is not empty) is called connected if it cannot be divided into two foreign (closed and not empty) parts (66, p. 197).

This concludes our preliminary presentation of mathematical-topological concepts. We shall not go further into the many, and often very complicated possibilities of topological relationships. The treatment of the psychological examples will give ample occasion to make the mathematical concepts familiar.

B. TOPOLOGY OF THE PSYCHOLOGICAL ENVIRONMENT

We shall start our discussion of topological problems in psychology by considering the psychological environment. In order to simplify the presentation we shall occasionally use concepts such as that of force which will not be defined until we discuss vector psychology.

CHAPTER XI

PSYCHOLOGICAL REGIONS, LOCOMOTIONS, AND COMMUNICATIONS

Coordinating Definitions

One should choose coordinating definitions in such a way that they hold without exception and are univocal. As far as possible we shall try to use reversible coordinations.

The Psychological Region.—Definition: To each part of the life space a region is to be coordinated.

Thus we have to represent as a region (1) everything in which an object of the life space, for instance a person, has its place; in which it moves; through which it carries out locomotions; (2) everything in which one can distinguish several positions or parts at the same time, or which is part of a more inclusive whole.

This definition implies that the person itself has to be represented as a region in the life space, further that the life space as a whole is a region.

The reverse of the definition of a psychological region also holds: everything that is shown as a region in representing a situation must be a part of the life space.

In determining whether we are dealing with one or with several psychological regions one can build on either of two facts: (1) one can characterize a region by its qualitative properties and can find out its relations of position by determining which regions are contained in others (have the relation to each other of part and whole, $X < Y$), how they overlap ($X \cdot Y \neq 0$ or $= 0$), which regions have common boundaries (b), and which do not ($b_X \cdot b_Y \neq 0$ or $= 0$). (2) One can build on psychological processes which connect different points (part regions) in the life space, for instance on locomotions. The locomotions cross or do not cross certain boundaries or other regions. This characteristic makes it possible, on the basis of the coordination of locomotion and path (about which see below), to make topological statements about the regions to which the points (part regions) belong.

The following are examples of qualitative characterizations of regions: ground which is easy or difficult to walk on; region of a forest; a region within which one may be seen from a certain point; a region of a certain color; the sphere of influence of a person; a social group; an occupation; a region in which certain actions are permitted. To determine the position of such regions, for instance the sphere of influence of a person, one can ascertain whether it overlaps the spheres of influence of other persons or groups and if so which; with which spheres of influence it has or has not points of contact (common boundaries).

In determining the position of regions one can use any bodily, quasi-social, or quasi-conceptual locomotion. We have already given sufficient examples of these locomotions. Locomotion makes it possible to make statements about the position not only of the region of departure and of destination but also of those regions which the locomotion crosses.

In psychological investigations one will have to use qualitative properties as well as locomotions in determining psychological regions and their positions. Sometimes the one, sometimes the other gives better results. On the whole the more reliable method of determining the topological characteristics of regions seems to be that of referring to locomotions.

The determination of a region, for instance by certain qualitative characteristics, does not in itself imply whether or not this region is a connected one. For instance, the region which corresponds to the property of a person or the region which corresponds to a certain social group has to be represented sometimes as a connected, sometimes as a non-connected region according to the actual distribution of the property or of the members of the group. Also in this point therefore the psychological concept of region agrees with that of mathematics (see p. 88).

Psychological Locomotion.—Definition: A path is to be coordinated mathematically to each psychological locomotion.

As stated above, one understands by path a part of a Jordan curve, *i.e.*, a curve which does not intersect itself. On the other hand psychological locomotion can, at least in a certain sense, pass the same place twice. In these cases locomotion would have to be represented by a curve which intersects itself. However, we shall generally speak of paths, since this is very unlikely to lead to error in the practical application of our concepts. (Besides, mathematics itself sometimes uses the concept of path in this more general sense.) It should be emphasized once more that in the following discussion we mean by locomotion not only quasi-physical but also quasi-social or quasi-conceptual locomotion.

The question arises whether this coordinating definition can be reversed. Such a reversal would take the following form: To each path in the life space corresponds a locomotion. However, there are cases in which one can connect mathematically points in two different regions of the life space, but when the corresponding locomotion can actually not be carried out. For instance, in our example the prisoner cannot carry out bodily locomotion from the region within the prison to the region outside. Nevertheless, in this case other objects in the life space of the prisoner can carry out such a locomotion and he himself can move in his thoughts from one region to the other. But it is at least conceivable that there can be regions in the life space into which even a conceptual locomotion cannot be carried out.

In spite of this difficulty it is possible to reverse the definition. We have already mentioned that it is sometimes easy, sometimes difficult to carry out a locomotion. A locomotion which cannot be carried out can therefore be thought of as an extreme case of difficult locomotion. Thus the concept of an impossible locomotion is entirely legitimate and even unavoidable. In other words one has to distinguish between the applicability of the concept of locomotion and its factual possibility. For our purposes we can state the definition in its reverse form as follows: To each path in the psychological life space corresponds a locomotion which can or cannot be carried out.

Analogous definitions are common in mathematics. Thus one usually defines function as follows: a is called a function of b if the value of a varies, or does not vary, with the value of b.

On the basis of these coordinating definitions of the psychological region and the psychological locomotion one can represent mathematically the topological relationships of an unlimited number of different situations. It is the task of the single psychological investigations to carry out these constructions. Here we only want to demonstrate by means of simple examples the general method of making such constructions.

Being Inside or Outside of a Region

We have explained that the different social positions differ significantly in space of free movement that is available to the person. A change in a person's social position can often be represented as a locomotion from one region to another. In general the fact that a person (P) is in a particular region (R)[1] at a given moment ($P < R$) is of decisive importance for his behavior. We shall explain that more exactly by means of a few examples.

An investigation has been made of the effect of social pressure on the behavior of children during meals.[2] It shows that one of the most important means by which the adult induces the child to eat an undesired food is to bring him into the "eating

[1] The region R may be defined in such a way as to include the objects located in it.

[2] This investigation (18) has been carried out in the nursery school of the College of Home Economics of Cornell University.

situation." If a particular kind of food is not desired, the otherwise unified action of eating usually breaks up into a series of separate steps such as: putting the hand on the table (*h*); taking the spoon (*sp*); putting the food on the spoon (*f*); bringing the spoon halfway to the mouth (*hw*); bringing it to the mouth (*m*); taking the food into the mouth (*i*); chewing

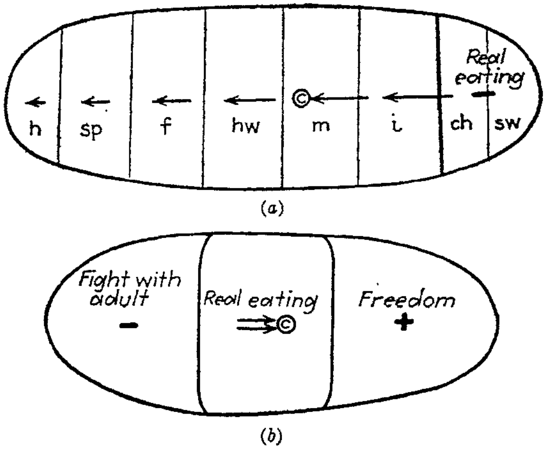


FIG. 16.—Situation of child facing disliked food. (*a*) before entering the region of real eating; (*b*) after entering the region of real eating. In this and the later figures the following symbols are used:

←—— force: the direction of the arrow represents the direction of the force; its length, the strength of the force; its point the point of application of the force.

←– – – – locomotion: the point of the arrow designates the place of termination of the locomotion, the other end of the dotted line designates the point of departure.

+ positive valence.

— negative valence.

(*ch*); swallowing (*sw*). These steps correspond topologically to a series of regions (*cf.* Fig. 16*a*). The procedure of the adult is sometimes to bring the child (*C*) step by step through these regions closer to the region of the "real eating" (chewing and swallowing). In doing so he usually meets with increasing resistance in accordance with the fact that with approach to the undesired action the repulsive forces (represented as arrows in Fig. 16) increase. However, as soon as the food is once in the mouth it is often not spit out, even when the adult has fed the child against its will. Instead the child goes on to chewing and swallowing the food.

One can show in detail that this change of behavior is brought about essentially by the fact that as the child enters the region of "real eating" his position and the direction of the field forces are entirely changed. When the child is in one of the preceding regions, for instance, when he holds the spoon halfway to his mouth, then a region of greater unpleasantness into which the adult tries to push him, still lies ahead. The adult therefore may have to exert great pressure to induce the child to make a locomotion into the disagreeable region. When the child is once within this region of real eating then the region which lies ahead of him is a more pleasant one of relative freedom (Fig. 16*b*). The child therefore often prefers a locomotion in this direction to spitting out, which is a locomotion in the direction of a disagreeable fight with the adult.

I cannot discuss further the various and often very complicated details of this situation. But I would like to point out that a similar technique is often used to force an adult to do something against his will, for instance in a political struggle. A social group may fight bitterly against the attempt to change its position. If however one succeeds in bringing about a *fait accompli* the group will accept its new position without resistance. This is one reason why the *fait accompli* is so dreaded in politics.

Another example: A two-year-old child does not want to stop playing and go in to wash his hands. The mother who knows that the child especially likes to wet the wash cloth in the basin for himself asks, "Do you want to wet the cloth or should I do it?" The child wants to do it and so he lets himself be washed without further trouble. The question has sufficed to transfer the child from the play situation to the washing situation and it begins to behave according to the requirements of the new region.

Again, Werfel (87a), in his book *The Forty Days of Musa Dagh*, describes how a group of Armenian peasants decide to defend themselves on a mountain against their common enemies. But they cannot come to an agreement about how to settle the property rights in their mountain camp. The priest finds a way to delay the discussion of the question. Later when they are once on the mountain the question of property adjusts itself according to the situation in a way which had been strongly rejected while they were still outside of the situation.

The importance of the act of decision for behavior lies mainly in the fact that by it the person changes his position.

These examples may suffice to make it clear how important it is whether one stands within or outside of a situation; expressed in mathematical terms whether one stands within ($P < R$) or outside of a certain region ($P \cdot R = 0$).

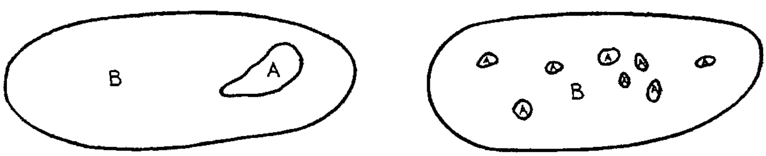
Why the region in which a person stands is so important for his behavior becomes intelligible when one realizes how great the change is which is brought about by a transition from one region into another, even within an otherwise unchanged life space. As a rule all relations of neighborhood are changed by such a transition: Regions which were before adjacent to the region of the person are no longer so and vice versa. Different locomotions are now possible and impossible. Even when the same regions are still attainable the course of the path which one must follow to reach them has changed because of the new point of departure. This usually means a change in direction and distance of other regions of the life space and thereby a change in the direction and magnitude of the forces which affect the person. But above all, it is important that the region itself in which he is located has a different character.

In short, the dynamic condition of a person depends in almost every respect directly on his position in a certain region. Methodologically therefore in almost every psychological problem one should give first place to the question of the region in which a person is at a certain moment, or what change of position is just occurring.

The Inner Structure of a Psychological Region

The topological properties of a region can be of very different kinds. In the following we shall discuss several pertinent questions in connection with simple examples of quasi-physical regions which we have already treated.

Determination of the Connectedness of Regions.—A simple example of the determination of the topological structure of a region by reference to possible locomotions is that of the "space of free movement." We have characterized the space of free movement as the totality of regions to which the person in question has access from his present position. Within this region of free movement the person can carry out locomotions from each point to every other without leaving the region, *i.e.*, without having to break through its boundary. On the basis of the coordination of psychological locomotion and mathematical path one can therefore designate the space of



(*a*) (*b*)
FIG. 17.—Minority group. (*a*) Unscattered; (*b*) scattered. *A*, The minority group; *B*, the majority group.

free movement of a person as a connected region. (The reader will remember that a connected region is defined by the fact that any two of its points can be connected by a path which lies wholly within the region.)

In determining the connectedness of a region in psychology it is not possible to observe the locomotions between all the parts of the region. It is sufficient to find out whether the essential parts can be connected by locomotions whose paths do not leave the region as a whole.

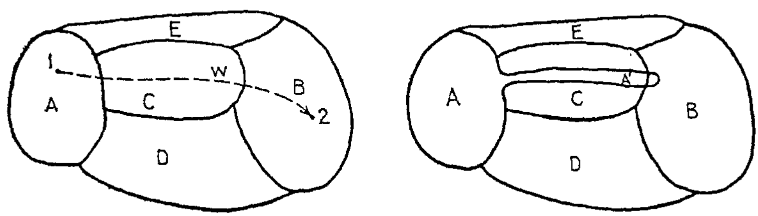
Nonconnected Regions.—For the behavior of a person who belongs to a minority group of a country the distribution of his group within the country is of great importance. The minority may be scattered, or it may live as a closed group. In the latter case the minority group (*A*) corresponds to a connected limited region within an enclosing region (*B*) which is to be coordinated to the other parts of the population of the country

(Fig. 17*a*). If the group is scattered it corresponds topologically to a nonconnected region (Fig. 17*b*). ($A^1 + A^2 + A^3 + \ldots$ is a not connected region.)

In so far as we are dealing with the geographical distribution one can demonstrate that the parts of the group are separated topologically simply enough by the fact that one cannot carry out a bodily locomotion within the region of the group from one of its parts to any other part. But it is also possible to determine the structure of such a group in regard to its social relationships. A group which is characterized by race or social position may live in a city without much social contact with the rest of the population. (One may think of the Negroes in New York; the nobility in a medieval Italian city, or any other exclusive social group.) This group would have to be represented as a connected region in so far as its social life is concerned if all of its parts can come into social contact with each other without the mediation of other circles which do not belong to this group. When social or business intercourse has to be carried out through members of other groups then the group has to be represented as regards its social or business life as a nonconnected region. One can see how it is possible in this way to determine in detail which parts of the region are connected and which are separated. Naturally the topological structure of the subparts can be determined in the same way.

One realizes how important the dynamic consequences of the connectedness or nonconnectedness of a group are if one considers how the behavior of its members is affected by the breaking up of a connected group. The breaking up of the Ghettos and the scattering of the Jews into smaller groups had as its result changes of behavior which are caused to a large extent by the change of the topological structure of the group. For a separation into nonconnected parts has dynamically the result of a weakening of the inner connectedness and the degree of mutual dependency (58, p. 182). It means further that, other things being equal, the surfaces of contact between this region and other regions are enlarged; also the magnitude of the forces which affect the group as such and its single members often are essentially influenced by the change of the connectedness of the group.

One must note in this example that it is possible to determine the topological properties of the social structure of a group on the basis of its social intercourse. It may seem doubtful whether we are really dealing with locomotions in this case; that is, whether in social intercourse the one person or the one group actually passes from its own region to the region of the other person or other group. There is no question that intercourse brings about a connection between the two regions. But this connection often has the character of a "communication" rather than that of a locomotion; one region shifts toward



(*a*) (*b*)
FIG. 18.—(*a*) Locomotion from *A* to *B*; (*b*) communication between *A* and *B*.

the other until a contact is made or one part of the region is so moved forward that it makes a bridge to the other region. Such a representation seems especially suitable if we are dealing not with intercourse between single persons but between groups.

Figures 18*a* and 18*b* show the difference between the two kinds of connection. In one case a one-dimensional path (w) leads from a point (1) within the region A to a point (2) of the region B. In the other case a two-dimensional arm (part region) A^1 reaches out from A to the region B so that it touches region B or partly overlaps it. ($A + B$ is a connected region.) In the second case we shall speak of "communication" (see p. 126). Locomotions and communications are in some respects of equal value for the determination of topological relation. This point will come up again in our discussions.

As we have mentioned above very different structures result if one takes as the basis for the determination of a region different kinds of locomotion or communication: the geographical structure of a social group can be very different from its social or vocational structure.

Multiply Connected Regions.—One can ask whether a given space of free movement is a simply or a multiply connected region.

Let us take as an example the space of free movement which was discussed on page 45. It consists of the regions of what one "can" do and what one "is allowed" to do. Such a space of free movement is, as a rule, a multiply connected region. The space of free movement is probably always surrounded by regions which the person is not able to enter. This would mean topologically that the space of free movement is a limited region (see p. 89). For it would lie entirely within a ring of inaccessible regions.

But it does not usually happen that all regions of the forbidden and the impossible belong to this surrounding ring and make up with it one connected region. In most cases there will be within the region of the allowed certain islands of the forbidden, *i.e.*, regions of the forbidden which are not connected with each other.

The nonconnectedness of these islands can be proved by reference to locomotions. For instance the regions which are characterized by such prohibitions as "You must not cross the road alone," "You must not copy in school," "You must not be impolite to a certain person," may usually be unconnected, especially when the one prohibition issues from the parents, the second from the teacher, and the third from a good friend. The transition from one of the forbidden regions to another forbidden region will usually be possible only by passing through the region of the allowed. This would prove that the two forbidden regions are not connected. At the same time it means that the space of free movement is in these cases multiply connected.

In some cases several regions of the forbidden are connected. This can have the dynamic effect that if once the boundary of the forbidden is crossed the person can pass from one part of this region to another with relative ease.

In such a situation other factors, especially the eventual weakening of the social fields which induce the prohibition, can play a role. This weakening is not caused by the topological connectedness of the different regions of the forbidden.

As a rule it is dynamically of no great importance to determine exactly how many times a multiply connected region is connected. It remains, however, important whether a space of free movement is frequently interrupted by islands of the forbidden or whether a space of equal extent is relatively free from such islands. A child may have a region of his own (a playroom, or a playground) in which he can follow his inclinations undisturbed. This means that this field is distinctly free from regions of the forbidden. If the child has to play in rooms which are also used by adults, he continually runs up against more or less extensive regions of the forbidden.

Limited and Closed Regions.—The topological structure of the situation in respect to bodily locomotions is especially simple in our example of the bathtub (see p. 42). Let us discuss the situation of the child *A*.

In the beginning *A* considers the whole bathtub as his space of free movement; since there is water in the tub, that is a material which does not hinder the locomotions in question (we shall limit ourselves to locomotions of the whole body). The space of free movement can be characterized mathematically in a very simple way: it is a connected limited region. The connectedness results from the possibility of locomotions from each point to every other on paths which lie wholly within the region. The fact that the region is limited is in this case manifested very simply by the surrounding rim of the bathtub.

But this does not tell us whether this space is a closed or an open region. Mathematically, as we have seen, it is characteristic of boundary points that they have no surrounding which lies wholly within this region. If one includes the boundary points in the space of free movement one thereby characterizes this space as a closed region. If one does not, the space of free movement would be an open region.

From a psychological point of view one can say: If one considers the edge of the tub as the real boundary of the space, then one is inclined not to include the boundary as part of the space, for naturally the child can only move about in the inner part of this region. This would mean that the space of free movement is defined as an open region. But, on the other hand, one could consider the points of the inner surface of the real edge of the tub, or perhaps the boundary line of the water at the edge of the tub, as the boundary of the space of free movement. Then one could not object to including the boundary in the space of free movement. In this case the space of free movement would be characterized as a closed region. We see that, from the point of view of psychology, it is irrelevant whether this space of free movement be defined as an open or as a closed region.

The question of whether a limited psychological region is to be characterized as an open one or as a closed one (that is, whether or not one should include the mathematical boundary curve in the region) seems to be a matter of minor importance. One must not forget that the open limited regions always have a boundary or a hull (66, p. 29).

Much more important are the dynamic characteristics of the boundary, for instance, its solidity. From the point of view of mathematics the edge of the tub is not really a boundary without thickness but is itself a region. We shall return to this question when we discuss boundaries.

In addition to the limitation of the space of free movement in our bathtub example the kind of connectedness is of factual importance. In the beginning the space has the character—so it seems at first—of a simply connected region. This follows mathematically from the fact that the boundary of the region is a Jordan curve. But this characterization of the space of free movement of *A* is not entirely correct. There is an "island" within this space, namely the boy *B*. The behavior of *A* would be quite different if *B* were not there. We do not want to discuss here the important difference between a permeable body of water and such a "thing" as a person. But it may be well to make it clear that one can include the second child in a mathematically consistent description without becoming psychologically unsound. B is a part of the life space of A and is therefore according to our definition (p. 93) himself a region. The body of B has the character of a thing (*cf.* p. 115), that is, a region which A cannot enter or cross. It is therefore not a part of A's space of free movement. The presence of B in the tub makes the space of free movement, if one wants to be exact, a doubly connected region. If B makes a connection between his body and the edge of the tub by grasping the edge e (Fig. 1*c*) with his arm B', B produces a cut (see p. 43)[1] through the space of free movement, but the space maintains its connectedness. A is still able to reach all points of the tub by making detours around B.

As a matter of fact B carries out his intention of limiting A's space in another way. He connects two points of the outer border by a line which he draws across the water with his finger, according to Fig. 1*b* (see p. 43). The originally connected space is divided by this cut into two regions. Only one part, the one in which A is, retains the character of a space of free movement, although it is now more restricted. The other part becomes a "power field" of $B(P_B)$ which A may not enter. The division made by the cut means at the same time a certain clarification of the situation ($P_B \cdot P_A = 0$). A is now sole master within his own region while, in the original region, his freedom of movement was hampered at least in the neighborhood of $B(P_B \cdot P_A \neq 0)$.

It is further important that A's space of free movement remains adjacent to that of B. If one wants to express this fact in exact mathematical terms one can say: the topological intersection of the boundaries of the two spaces of free movement (b_A and b_B) is not empty ($b_A \cdot b_B \neq 0$). This is an example of a determination of the adjacency of two regions without reference to locomotion.

One might raise the question whether the body of A himself should not be treated as an object in his own space of free move-

[1] He makes a connection between different boundary points of the space of free movement by means of a path which lies within this space.

ment. This would mean that this space is at first a threefold, later a twofold connected region. In principle there is no objection to such a representation, and there are cases in which it is even advisable from a psychological point of view. On the whole it will be better to represent the own person as point or region which moves about within the space of free movement but which is not a foreign region in it. We shall later return to the question of the representation of the person in the life space.

In regard to each region which has the character of a thing, one can ask whether it makes up a part of the space of free movement itself or whether it lies within this space without belonging to it. Psychologically this question is only meaningful when we are dealing with sufficiently large or otherwise significant objects that may for instance be obstacles to certain locomotions. The answer to this question will depend upon the special character of the situation and the special kind of locomotion.

Representation as Path or as More Than One-dimensional Region

It is sometimes doubtful whether one should represent a certain fact in the life space as a path or as a region. We have already met this question: Social intercourse can be a locomotion which must be represented as path or it can be a communication which has to be represented as region (or part region).

Mathematically the concept of region also includes one-dimensional manifolds (see p. 88). A path therefore can be understood as a region no less than those parts of the life space which we have represented by two-dimensional manifolds. The question how many dimensions the life space has will be discussed later (see p. 193). In any case there is a great difference between a two- or more-dimensional region in which or through which a path may take its course and these paths themselves. The very fact that they differ in number of dimensions is important. It is of still greater significance psychologically that (in our case) paths are coordinated to locomotions, *i.e.*, to processes in time, but psychological regions to the areas in which these processes occur. It is therefore an important question whether one should represent a certain psychological fact as a path or as a two- or more-dimensional region.

In the example of the child who had to eat something which he did not like we have represented the actions which lead up to eating and eating itself as regions through which the child had to move. We have further spoken of a person's occupation as a region. In experimental work too we have found that it is sometimes useful to represent activities as psychological regions in the life space.

It may seem surprising that an action can be characterized psychologically as a region, and moreover as a region of the psychological environment rather than of the person. We do not claim that actions have always to be characterized as regions. Generally one thinks of an action as an event, a process in time. This process can have the character of a quasi-physical, quasi-social, or a quasi-conceptual locomotion and is then to be represented as a path.

However, it seems advisable, in certain cases, to represent actions as regions. For instance a child may be busy playing with its dolls. The mother calls it to eat its supper or to go to bed. The difficulties which are typical in such cases are connected with the fact that the child has to leave the region of a certain occupation and has to enter the quite different region of eating or sleeping. Or again, if a man leaves his business to go on a vacation, or if he changes from one occupation to another, we are dealing with a locomotion across a boundary. The actions of playing and eating have, at least at this moment, the character of regions in which the child is located, out of which or into which he has to go; that is the character of regions in the environment.

The regions of action correspond to other regions in the life space also in that one can move about in them, that they can be parts of more inclusive regions, and that they can contain part regions. A child who has to solve a problem in arithmetic for his school work may move about within the region of the problem until he has found the solution. He may then leave the region and go to another task in arithmetic until he has finished that one as well. From there he may go on to other school work, for instance the preparation of a French exercise. The appearance of a playmate may cause the child to leave the region of school work before he has finished and to go and play with his friends. In this example the arithmetic problem is part of the whole region of school work and includes part regions which correspond to the single problems. The single problem can include part regions which correspond to the single mathematical operations.

The locomotions within such a region can have a purposive character. In the case of the arithmetic problem the solution of the task is the goal which controls the locomotion. In other cases the process within the region of activity may have less the character of striving toward a goal than that of staying or moving about within a region. This is true of such activities as dancing, the infant's thumb sucking, the child's play with dolls. One's regular work also can be purposive to different degrees. For the unskilled laborer work has more the character of a field within which he remains than it has for the skilled workman (50).

In many cases it may be doubtful whether one moves at all while one is performing an action in a given region of activity. To sit quietly and muse may be considered as staying within the same region and even at the same point. At least one is not engaged in any swift locomotion.

The temporal process of locomotion, independent of its velocity, is a change in the position of a person whose course can be represented by a path. This path as we have mentioned before (see p. 95) must not be thought of as representing a part of the life space as it exists at a given moment but rather a change of position within a field which otherwise remains sufficiently constant. Such a representation is essentially an abbreviated representation of a sequence of situations and is therefore possible only within periods of time in which such a constancy exists.[1] The actions on the other hand which are to be represented as regions in the life space are coexisting manifolds which possess a certain structure and a certain degree of differentiation.

Representation as Point or as More Than One-dimensional Region

The relation between paths and regions becomes clearer if we consider the relation between points and regions in the life space. It is sometimes doubtful whether we should represent a psychological fact as a point or as a more-dimensional region. (Mathematically a point is a o-dimensional region.)

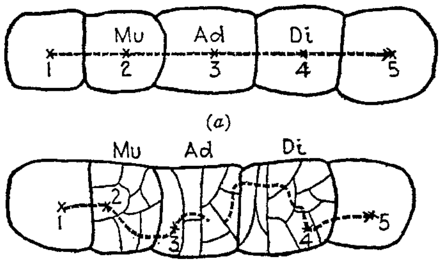
To a locomotion we have coordinated a path, *i.e.*, a connection between two points. Psychologically also it seems entirely correct to say that a locomotion leads from a beginning to an end point. This end point is often a goal toward which a person strives. As a matter of fact one can represent goals as points. On closer consideration however one finds that a psychological goal is always a region. This is true of the apple toward which the child strives[2] as well as of the occupational aim of the youth who wishes to become a physician. Goals are not points but regions into which a person would like to enter or in relation to which he wishes to have a certain position.

Also if one tries to characterize the intermediate points of such a path, one is often faced with facts that one cannot represent as points, but only as regions. One can represent the process of working an arithmetic problem in which one has to perform first a multiplication, then an addition, and finally a division as a locomotion to which one coordinates a path from a

[1] We usually represent a locomotion in this case as a broken line which starts at the moving region and whose end point is characterized by an arrow. The moving region is shown in the position at the beginning of the locomotion. Such a representation makes it possible to distinguish between force and movement.

[2] It is usually not correct to designate the material object itself as the goal. The goal is usually an action or a state, for instance the eating of an apple or the possessing of an object.

beginning point (1) by way of certain intermediate points (2, 3, and 4) to an end point (5) (Fig. 19*a*). This implies that one has to represent multiplication as a region of action (*Mu*) which the path crosses and not as a point. The progress from multiplication to addition (*Ad*) and division (*Di*) means that a person has passed from one region to another. The question arises whether one can determine exactly the points of the path within these regions. These points are at first only characterized by the fact that they lie in such and such a region. We can determine their position more exactly only if we succeed in



(*b*)

FIG. 19.—The maximum exactness with which the position of a point can be determined depends upon the degree of differentiation of the region in which it lies. (*a*) Relatively undifferentiated regions; (*b*) relatively differentiated regions. *Mu*, multiplication; *Ad*, addition; *Di*, division; 1, 2, 3, 4, 5, different points.

characterizing subregions within the larger regions. It may be possible to divide the region which corresponds to a complicated division into subregions according to the part operations involved. The topological relationships of these part regions are determined to a certain degree by the sequence of these operations. Then one can say that the path which is to be coordinated to the action passes through the subregions in such and such a sequence (*cf.* Fig. 19*b*).

It is important for the determination of the life space that the position of a point cannot be characterized except in terms of its position in such and such a region. The exactness of this determination depends, in the single case, upon the extent to which one can distinguish subregions within the region in question.[1]

[1] The same is true of the determination of real points in physics.

With this fact is connected a second: if it is true that one can determine the position of a point only in so far as one can divide the regions in which it lies into part regions, it does not make any difference whether one speaks of the beginning point and end point of a locomotion or of its beginning region and end region. The point becomes thereby, for our consideration, equivalent to the region in which it lies and which can be no further differentiated. In other words: in psychology the point is equivalent to a region which is not structured into parts.

This means for our representation two things: first, if a psychological fact is represented by a point, one is justified in considering this point, provided one is to examine it more closely, as a more-dimensional region; second, one can sometimes represent unstructured regions in the life space as points.[1]

In some cases one can make use of these facts in the representation of the person. We have seen that it is always necessary to represent both person and environment in the life space. We also have mentioned that one has to distinguish within the person certain strata and regions. The person himself is therefore a more than zero-dimensional region. For certain problems however one can represent the person in a first approximation as a point for the following reason: the person is a strongly unified whole. When we are not dealing with the dynamic differentiation of the person into part regions we can consider the person as a single system and can represent it as an undifferentiated region or as a point in the sense explained above. One can use such a representation especially in those cases that concern the locomotion of the person as a whole. The same representation is valuable in treating forces which affect the locomotion of the person. To be sure it is admissible only when the point of application of the forces within the person is not important; this means again when the person in this respect can be thought of as an undifferentiated whole.[2]

[1] Also from a purely mathematical point of view a point is an unstructured region.

[2] In these cases we usually represent the person in our diagrams as a limited, simply connected, and undifferentiated region.

For more exact representation one will always have to show the person as a differentiated region and the specific point of application of the force.

Locomotions of a Surrounding Field

Occasionally there are definite changes of position of the person in the surrounding field when the person does not carry out any active locomotion himself.

Some of these cases are very simple. A man may have been swept along by external social changes. The social position of a person *P* may change greatly as a result of gain or loss in the influence of his family or of the business concern with which he is identified. History shows how closely the rise or fall of single persons is bound up with the fortunes of whole groups. A change of a person *P* in his environment *E* often appears as active locomotion of *P* when in reality this change is not a result of a movement of *P* in relation to his immediate social environment, *i.e.*, the group *G*. In reality it may have been brought about by a movement of the group *G* in relation to the whole field.

It is easy to characterize social locomotions of this kind. They correspond to the change of place of a person who is in a moving train. Such cases offer no special conceptual difficulties. One only has to take into account the relative character of all movement.

More surprising than this kind of locomotion are other cases. We can offer an example from our moving-picture material. A two-year-old child *C* who still has trouble in walking up- or downstairs without support wants to place his ball on the landing. In order to do so he has to go up three steps. Topologically we could represent the initial stage of the situation as follows (*cf.* Fig. 20*a*). Between the goal *G* and the child *C* there is a barrier which consists of the following zones: climbing the first step (*c*1), climbing the second step (*c*2), the third step (*c*3), and finally going beyond the edge of the landing which is still a danger zone (*dz*) from which the ball may roll back. Let us assume that the child has already picked up the ball (*Ba*).

(The child *C* and ball may be represented as regions which have a partially common boundary.)

The child succeeds in bringing the ball up all three steps to the danger zone *dz* (Fig. 20*b*). Then he drops the ball and it rolls down again. Thereby the following situation comes into being (Fig. 20*c*). The goal is suddenly moved to a distance. There are now more regions between the child and his goal than in the original situation. In order to reach his goal the child

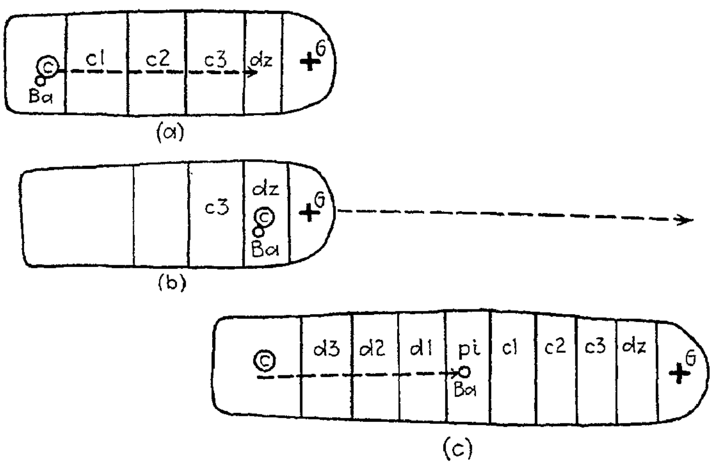


FIG. 20.—"Ground moves under one's feet." (*a*) Child starts to climb the steps carrying a ball; (*b*) the goal is nearly reached; (*c*) the ball has rolled down. *C*, Child; *Ba*, ball; *G*, goal. *cl*, Climbing up the first step; *c2*, the second step; *c3*, the third step. *d1*, Climbing down the first step; *d2*, the second step; *d3*, the third step. *pi*, Picking up the ball; *dz*, danger zone.

must now go through the following regions: He must climb down the third step (*d3*), the second step (*d2*), the first step (*d1*), and pick up (*pi*) the ball. Then he must again climb up the first step (*c1*), the second step (*c2*), and the third step (*c3*) with the ball in his hands, and he must take the ball beyond the danger zone (*dz*) on the landing.

Without doubt there occurs a significant change in spacial relations of *C* and *G*. Since all locomotions can be determined only relatively there is no reason for not speaking of locomotion in this case. The causes of this locomotion are essentially different from those of the active movement of the child between the first and the second stages. Then it was the ball which separated itself from the child and carried out the locomotion to *pi*. At the same time however the spacial relationship of *C* and *G* underwent a marked change as a result of the locomotion of the ball. Since *C* did not bring about this change of position by active movement on his own part, and furthermore since he was not passively moved by another person, he might have the feeling that the "ground moved under his feet." Such an event may well be characterized as a locomotion of the surrounding field in relation to the person. Locomotions of this kind are often accompanied by other changes in the structure of the field.

As we have mentioned above there seems to be no doubt that in psychology as in other sciences locomotion can be determined only relatively, *i.e.*, as a change of position of one region in relation to others. One can therefore raise the question whether it means anything to distinguish the movement of the person from an opposed movement of the surrounding field. The time has not yet come when psychology can enter into a discussion of this problem which corresponds to the question regarding the relativity of movement in physics.

Thing and Medium

Psychological regions of the environment can show very different dynamical properties. They can offer either great or slight resistance to locomotion; they can attract, can be neutral, or can repulse; they can represent living beings or objects; they can exhibit any degree of fluidity or elasticity; they can react differently to different influences. We shall discuss a few of these properties later. At this place we shall only emphasize a certain kinematic difference.

If one represents an activity as a region one represents it at the same time as a "medium" (32). A goal toward which one moves, on the other hand, has not the character of a medium but of a "thing." The person who moves also is such a thing, as is likewise another person in the life space.

From a kinematic point of view one may call a region a medium (in opposition to a thing) if movements can be carried out in or through it (*cf.* 32).

There is no question about the thing character of a great number of quasi-physical objects like a ball, a table, a tool, etc. The medium character of certain other regions is also clear, for instance the air for the flyer. In other cases of quasi-physical regions the character is not so unambiguous. It can change with changes in the situation. A hut in the mountain has the character of a thing as long as one is trying to reach it from a distance. As soon as one goes in, it serves as a region in which one can move about. Similarly, a bucket may at first impress a child with its object character. Later, in the course of a game with water it may become a region in which different kinds of locomotions can be carried out. The fact that it is not a locomotion of the whole person but only a locomotion of a hand is irrelevant for our consideration.

This example may show that, as far as the medium character of a region is concerned, one must think not only in terms of locomotion of the whole person but also of any other regions. It can well be that what is a thing for the locomotions of the whole person is a medium for the locomotions of other objects. For instance a toy bank is a medium in regard to the coins that are in it.

A somewhat different example of the fact that an object can be at one time a thing and at another a medium is the difference in the psychological import of an object for a child and for an adult. A barrel may be a thing for an adult while a child can move about in it as in a medium. This is one reason why the same outer world may have different meaning for child and adult.

In regions other than the quasi-physical it also often happens that a thing changes into a medium or the reverse. The home work that a pupil has to hand in within three weeks, and an operation which a person must undergo at the end of six months have the character of a thing for the person concerned. The same holds for many events or actions which are temporally distant and which are undifferentiated regions in the life space. Gradually as the time for the operation or the final date for handing in the home work comes nearer it becomes something that one must "go through." We may say that there is a clearer differentiation of its regions into subregions in which one can move about, and that the medium character finally becomes entirely evident as one enters the region.

One thinks generally of a medium as a region which offers no resistance to a movement, whereas a thing seems something dynamically compact and solid. However, one must realize that regions may offer all possible degrees of resistance. There are regions which can be crossed but which still act as obstacles to movement. For bodily locomotion for instance, a thick underbrush is a medium which offers definite "friction." This friction can increase until it is impossible to advance farther. Then one is no longer dealing with a space of free movement but with a boundary of this space. This example shows clearly that there are all possible transitions between the dynamical properties of thing and medium.

CHAPTER XII

BOUNDARIES OF PSYCHOLOGICAL REGIONS

Definition and Determination of Psychological Boundaries

In making use of the mathematical definition of a boundary one can give the following definition:

Definition: We shall designate as the boundary of a psychological region those points of a region for which there is no surrounding that lies entirely within the region.

One can determine existence and position of a certain boundary in the psychological life space in different ways according to the nature of the case under consideration. It is possible that one can survey, for instance in the quasi-physical field, a number of regions simultaneously and determine the boundaries between them without difficulty.

For dynamical problems psychological locomotion plays again an important role. However, it would not be correct to determine, as perhaps seems most simple, the boundary points as those points "beyond which one cannot go" without leaving the region. For such a determination would presuppose the concept of direction, which is not admissible in topology.

It would be less objectionable to designate as the boundary points those points of a psychological region which one can not encircle without leaving the region. In the case of the space of free movement one could for instance say: Its boundary points are those points which the person can touch but not encircle. But this determination also is not unassailable. For we find cases in which one can encircle boundary points of a region without leaving it; namely, when we are dealing with boundary points toward a second region which lies like an island entirely within the first region. If one can exclude this possibility in a given case the procedure is correct.

In carrying out a locomotion the experience of crossing a boundary is often a clear one. This is for instance the case when one climbs over a fence or enters a strange house for the first time; or, to use an example of a quasi-social locomotion, if one is admitted to membership in a club by some special ceremony. Thereby the position of the boundary is quite accurately determined. However, there are cases in which one can establish with certainty that the locomotion has proceeded from one region into another one, although the crossing of the boundary does not become evident as a special event during the locomotion. For instance, one can gradually pass from one social circle into another. A path may lead from the mountains into lower hills and on to a plain, or from a great city through more and more open suburbs into the country and it may be impossible to describe definite boundaries between these regions. The same is true for all gradual transitions between two regions. For instance, it can happen in conversation that one is not even aware of a "gradual transition." That the person has passed the boundary can then be inferred only indirectly from the fact that he is in another region. In these cases it even remains doubtful how many boundaries and intermediate regions the locomotion has crossed.

Sharpness of a Boundary; Boundary Zones

Even when the transition occurs gradually one can make statements about the position of the boundaries. One can in such cases think of the boundary as a boundary zone, that is, not as a one-dimensional but as a more-dimensional region.

The position of this intermediate region is determined above all by the fact that it intersects the path which corresponds to the locomotion concerned between its beginning point and end point. It is often possible to determine the position of the intermediate zone still more exactly if one can locate all points of the path which definitely lie in the beginning region or end region. By such a convergent approach one can often determine the position and width of the boundary zone with a high degree of accuracy. According to the width[1] of this zone of gradual transition we shall talk of more or less sharp boundaries. We have already pointed out that on closer examination we find that all real boundaries of psychological regions are not curves or surfaces without thickness but that they themselves are more-dimensional regions. However, there are considerable differences in this respect.

In the example of the bathtub the edge is so thin that it has psychologically not the character of an enclosing region but

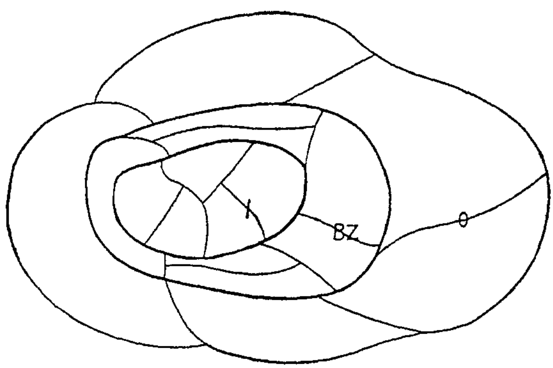


FIG. 21.—Boundary zone between two regions. *I*, inner region; *O*, outer region; *BZ*, boundary zone.

that of a boundary without thickness. To some degree the same may be true of the walls of the prison. But if there is a ditch or a barbed wire entanglement behind the wall, the boundary assumes much more definitely the character of a zone. The same is true if the prison wall is protected by machine guns. Then in making his escape the prisoner must also pass the danger zone of the machine guns. In such cases the inner region (*I*) is separated by a boundary zone (*BZ*) from the outer region (*O*) (*cf.* Fig. 21). Instead of two regions (*I* and *O*) and a boundary (*B*) one can then speak of three regions (*I*, *O*,

[1] The concept of width goes beyond topology. However sometimes one can make use of the fact that one boundary zone lies entirely within another. The transition to a greater exactness of determination of boundaries has meaning also from a purely topological point of view because it can be based directly on the relation of "being-contained-in."

and BZ) which have a definite topological relation. Each of these regions can of course have part regions.

Definition: We call a boundary zone between two regions (m and n) that region (BZ) which is foreign to m and n and which has to be crossed by a locomotion from one of them to the other ($m \cdot BZ = 0; n \cdot BZ = 0; m + n + BZ$ is a connected region).

In the example of the prison the boundary zone is a connected region which separates as a whole an inner and an outer region and whose border consists of two separate Jordan curves.

If the width of a boundary zone is psychologically irrelevant, one can represent it as a one-dimensional boundary. Such a representation is permissible as a first approximation even when we are really dealing with a boundary zone. The relation between boundaries and boundary zones is similar to that between points and more-dimensional regions. As we have seen one can sometimes use a point to represent undifferentiated regions. It is clear that in a similar way a boundary can stand for a boundary zone which is not differentiated in depth. One can always proceed later to a more exact representation by means of a boundary zone.

It often happens that in the course of events the character of a boundary changes, for instance when the person concerned approaches the boundary or begins to think about it. Then boundaries which have at first the character of a one-dimensional curve may later differentiate themselves into boundary zones. The reverse may also happen. We found a similar dependency on psychological distance when we discussed thing and medium.

Finally, it can happen that regions assume the character of a boundary zone because a path which connects two other regions has to cross it. An example: a child who is eating his dinner must prepare his lessons before he can play. In this case the lessons assume the character of a boundary zone.

We mentioned that one can think of unsharp boundaries as boundary zones whose width corresponds to the degree of unsharpness. Sharp psychological boundaries correspond best to mathematical boundaries. On the other hand not every boundary with pronounced depth implies an unsharp transition. An example from social psychology may serve as a demonstration. While the boundary between different economic classes is in general relatively unsharp and is characterized by a gradual transition, the boundary of some social groups such as an exclusive club is sharply defined. This means that for every person it is clearly determined whether or not he belongs to the group. Nevertheless the boundary of such a group can have the character of a boundary zone. In order to join the club for example it may be necessary to have one's name put on a waiting list in advance. Sometimes several such stages are prescribed. Therefore the existence of a boundary zone does not necessarily lessen the sharpness of the boundary, for the boundary zone itself may be a region which is clearly structured and sharply defined as to its boundaries.

The degree of sharpness of the boundary seems to be of great importance for the inner structure of a social group, especially for its homogeneity and for the processes within it. An example is the difference between social life in the United States and in Germany. It seems to me to be one of the most important characteristics of the social structure of the United States that many socially relevant regions are more sharply bounded than in Germany. This can be observed in small things of the daily life, as well as in the political, vocational, and social structure of the country (for instance, in the importance which one attaches to punctuality, or the exactness with which a schedule is made out; in occupational specialization; in the sharpness with which governmental responsibilities are defined (55)). Pedagogically it is of great importance whether the regions of play, eating, sleeping, and working in the life space of a child are clearly and sharply separated or whether there exist broad regions of unclear transitions. The same is true, as we have mentioned, of the regions of the permitted and the forbidden, of freedom and coercion. Unclear zones of unsharp transitions lead more often to tension and conflicts. When dealing with this question also one will have to take into account the special characteristics of the situation. Incidentally in pedagogical matters too, especially as regards what is forbidden and permitted, the boundaries of the regions seem to be comparatively more sharply defined in America.

One must not confuse sharpness of boundaries with their solidity (see p. 124). The life space of the child for instance corresponds, as we shall see, to a relatively fluid medium. Nevertheless it seems to show an especially strong tendency to sharp boundaries. It may perhaps be a general rule that unsharp boundaries are more apt to be found in a relatively solid than in a relatively fluid medium. Certainly individuals differ in respect to the sharpness of the boundaries within the life space and in the tendency to avoid unsharp boundaries.

Dynamic Properties of Psychological Boundaries

Boundaries as well as boundary zones can have very different dynamic properties.

Barriers.—Dynamically the principal difference between the prison wall and the edge of the bathtub is that the one is much more easily crossed than the other. In general the resistance which a boundary offers to being crossed is very important. This resistance can have all values between almost zero and infinity. This is true for unsharp as well as for sharp boundaries. If the prison is surrounded by an open field, there is a gradual transition between the danger zone in which an escaping prisoner could be reached by a machine gun and more distant regions which are less dangerous. In other cases, for instance if there is a dense wood just beyond the open field, the region of greatest risk is sharply bounded. In both cases the boundaries of this region need not offer any special resistance to the bodily locomotion of the prisoner. It may be possible to go through the gate that separates street and garden without noticeable resistance. In general there is no difficulty in crossing the sharply defined boundary between sidewalk and street pavement. Also the experience of a transition is not necessarily limited to boundaries which are difficult to cross but can also occur with easily passable boundaries, especially when the boundary is sharp and the two regions sufficiently different in quality.

At the other end of the scale of difficulties are the impassable boundaries. For the man who cannot swim, a river may be impassable. Each insurmountable object is an example of an impassable boundary for quasi-physical locomotion. The physical properties of impassable boundaries can be of very different nature. Spring floods or an unusually swift current may make a river impassable for the swimmer; the speed of a railroad train means a strong boundary against any attempt to board or leave the train while it is in motion.

Like the boundaries of the quasi-physical regions those of the quasi-social regions can be different in regard to their dynamical properties. It may be very difficult for an outsider to gain access to a certain social group. However, in social regions too, the boundary does not necessarily imply difficulty for locomotion. Many clubs represent well-defined groups, although entrance into them may offer no real difficulties. Also the boundary of a crowd, for instance of spectators or of people who stage a demonstration, may easily be crossed. The strength of the boundary can suddenly increase. If, for instance, during a demonstration the crowd is attacked from all sides, it may become difficult to separate oneself from it.

Definition: We shall call boundaries (boundary zones) which offer resistance to psychological locomotion "barriers." We shall speak of barriers of different strength according to the degree of their resistance.

We shall continue to use the concept of boundary in a purely topological sense. The term "psychologically real" boundary therefore does not imply defined dynamic properties.

Our examples have shown that a barrier can offer different kinds of resistance to locomotions. It can have very different degrees of "solidity," different degrees of "rigidity" or "elasticity" (see p. 159). It can oppose a locomotion at a certain point like a fence. (This is true of the edge of the river for the person who cannot swim.) Or it can have the character of a boundary zone, which offers resistance although it does not make further progress impossible. (This is true for instance of a piece of land which is difficult to cross.) In this case one can speak of "friction." Finally the barrier can have the character of a more or less permeable membrane.

In discussing the space of free movement we have already mentioned that the boundary of a psychological region can be easily passable for one kind of locomotion and impassable for another. The strength of a barrier is therefore always to be defined in relation to a certain kind of locomotion. It is not only different for quasi-physical, quasi-social, and quasi-conceptual locomotion, but also for different kinds of quasi-physical locomotion (swimming, driving, looking). The following example from Mrs. Lindbergh's North to the Orient (61, pp. 220–221) gives a very impressive description of such a difference. Colonel Lindbergh and two physicians were leaving a crowd of starving Chinese in a flood area to whom they had tried to take medical supplies: "Looking down on the spot they had just left, the men in the plane were acutely conscious of the miracle of their escape. A moment before they had been down in that crowd of starving people, some of whom might live until spring; many would die before the waters receded. Now, headed for Nanking, safety, food, and shelter were as assured to the fliers as in their own homes. Separated from those desperate people below only by a few seconds in time, only by a few hundred feet in distance, they were yet irretrievably removed in some fourth dimension. The two worlds were separated by a gulf which, although not wide, was deep, perilous, and unbridgeable. At least it was unbridgeable to the owners of the sampans. The fliers had crossed over from one world to another as easily, as swiftly, as one crosses from the world of nightmare to the world of reality in the flash of waking.

"They had a gun; they had a plane—powerful as any genii to be summoned from a magic lamp. And yet, magic rests on a knife-edge—a lamp, a tinder-box, an 'open sesame.' It is a hair-bridge between captivity and escape; safety and danger; life and death. The pull of a trigger, the press of a switch—

without these, the three magicians flying back to Nanking would have been simply three people in a starving, dying, and devastated land."

The difficulties of crossing the boundary are not always the same in entering a region and in leaving it. Thus we have to recognize that the dynamic characteristics of a boundary may be different for locomotion in different directions.

A boundary does not necessarily have the same strength at all points. It often has parts which one can pass easily and others which offer great difficulty. The fact that different sectors of a boundary can have different dynamical properties is important, for instance, for the problem of detour.

The boundaries of social regions too do not usually have the same solidity at all points. Success or failure in entering social groups often depends upon whether one finds the correct approach. The successful impostor is especially clever in choosing his points of approach.

It may be mentioned that even a single person can be thought of as a social region in the sense discussed above. Boundaries of different degrees of strength correspond dynamically to the different degrees of accessibility which distinguish individuals from each other (55). Again the boundary of a person is not equally strong at all points. In trying to make a contact it is important to find the right approach.

Boundaries Which Affect Communication.—In determining boundaries of psychological regions and their dynamical properties we cannot limit ourselves to the consideration of psychological locomotion. As we have already said, communications are of no less importance for quasi-physical than for quasi-social and quasi-conceptual fields.

Definition: By degree of communication of a region *a* with a region *b* we understand the degree of the influence of the state of *a* on *b*.

In so far as communications are concerned we designate psychological boundaries in general as dynamic "walls." We speak of the strength of a wall in the sense that a high degree of communication corresponds to a weak wall. For the concept of barrier the difficulty of a locomotion across a boundary is relevant; here we are dealing with boundaries which affect the influence exerted by the state of one region on the state of another.

In treating the topology of the person we shall have opportunity to discuss more in detail the problems of communication of regions. However, it must be emphasized that these problems have bearing on the psychological environment as well. For instance, as we have mentioned, the degree of communication between different social groups is of essential importance.

The fact that a region *a* is in communication with a region *b* does not, according to our definition, imply that *b* is in equally close communication with *a*. As we have seen, the strength of the resistance which a barrier offers to locomotion can vary according to the direction of the locomotion. Likewise the strength of a dynamic wall can have a different value for processes of communication from *a* to *b* and from *b* to *a*.

An example is the communication between two persons when one looks at another. If a mother looks her child in the eye when she is trying to induce him to carry out a certain action or to emphasize a command, the looking is certainly a real process which can have a great influence on the course of events. One could think of representing "looking at" as a locomotion. It certainly is a kind of intercourse. However, the objection can be made that it is not the whole person *A* which carries out the locomotion to *B*. Yet the "looking at" brings *A* into contact with *B*. "Looking at" in this respect corresponds for instance to a touching of *B* by stretching out the hand. As a matter of fact "looking at" can be a direct substitute for the touching of the child by the mother. "Looking at" therefore would have to be represented as a reaching out of an "arm"; or topologically as locomotion of a part A' of A in such a way that the part touches B without separating itself from the main region of A. ($A + A' + B$ is a connected region.) (Fig. 22*a*. This representation agrees with that of social intercourse, Fig. 18*c*, which we have discussed on page 102.)

The mother establishes the contact to influence her child, that is, to change the state of the child in a certain way. The "looking at" is therefore an establishment of contact for the purpose of a communication in the defined sense. This communication occurs only if the mother succeeds in catching the child's eye. Mother and child must look at each other. The child often tries to avoid the influence by dodging the mother's glance. He avoids getting into communication with the mother. Sometimes he may look at the mother in an impudent way. In these cases the child is closed to the influence of the

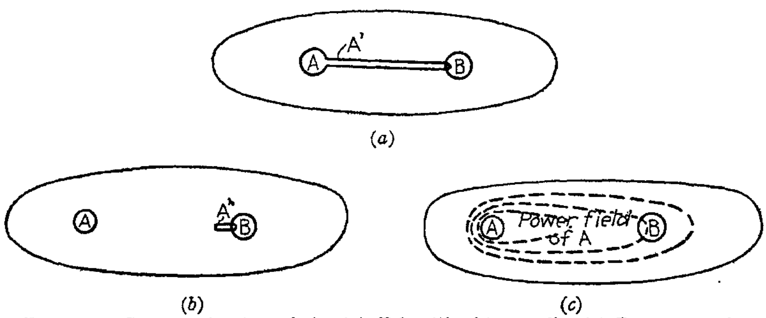


FIG. 22.—Communication of *A* with *B* by "looking at." (*a*) Represented as "arm" of *A*; (*b*) represented as separated region of *A*; (*c*) represented as power field of *A*. *A*, person looking at *B*; *A'*, region corresponding to "looking at."

mother's glance in spite of the fact that he looks at her; there exists an inner wall which more or less blocks the influence of the mother. The child wants to preserve his own state and even to influence the state of the mother. The opposite case is realized when the child looks at the mother ready to carry out her least wish.

A similar situation exists if one looks at a work of art in an uncritical manner and gives himself entirely to it. In this case the onlooker actively establishes a communication by "looking at" and then assumes an attitude of complete receptivity; that is, he tries to make the dynamical walls between himself and the work of art as weak as possible and to allow the influence to proceed toward his own person. If the glance only plays across a number of objects, the communication is usually very weak.

Thus in all these cases "looking at" establishes a contact. But the direction in which the influence occurs (whether mainly from *A* to *B* or from *B* to *A*) and the degree of communication are very different. It depends on the state of the person who looks, on the state of that on which he looks, and on the kind of looking. The degree of communication always depends on the properties of the communicating regions and the kind of communication.

Another fact of general methodological importance may be pointed out in connection with this example. One could object to the representation which we have given on the grounds that the bridge between *A* and *B* which is established by looking has not the character of a continuously solid part of *A* as is the case when *A* reaches out his hand toward *B*. It should therefore not be permissible to represent "looking at" as a reaching out of an arm. Without doubt there are essential differences between these two cases, and one could think of representing "looking at" according to Fig. 22*b* as a touching of *B* in which there is no continuous connection between *A* and *B*. *A* would then throw his glance to *B* like a ball (*A'*). This ball however has not the character of a solid body but rather that of a force. Indeed direction and kind of looking are directly related to what one can call the power field, the sphere of influence of a person and what one can represent psychologically as a field of forces. Wiehe (91) found that these fields of forces are in general stronger and reach farther in front, in the direction of the person's glance, than behind him. The dynamic nature of the arm which reaches from *A* to *B* can probably best be thought of as that of a field of forces (Fig. 22*c*); "looking at" can be considered as a change of the position and intensity of this field of forces. However, the topological correctness of our representation is not thereby impaired. There is no reason why one should not also treat fields of forces as regions and represent their relations of position, in a first approximation, by topological means. Certainly the dynamic nature of such regions needs a characterization which goes beyond topology. This, however, as we have said repeatedly, is true of all psychological regions.

The question whether "looking at" is to be represented as an arm A' according to Fig. 22*a* or as a region A' which is separated from A according to Fig. 22*b* also must be asked if one considers "looking at" as a field of forces. There may be cases in which Fig. 22*b* is preferable. In general however Fig. 22*a*, which corresponds to Fig. 22*c*, may be more correct. For it is possible to impair the communication that exists between the mother and child who are looking each other in the eye by interference at any point of the immediate region between A and B or to block it entirely by erecting an opaque wall between them.

Boundary Zones Which Can Be Passed Only with Difficulty.—As we have mentioned, a psychological barrier need not have the character of a thing but may be like a boundary zone that can be crossed only with difficulty. Since one usually understands by the term "barrier" a solid thing-like object this group of psychologically real boundaries may be treated specifically in this place.

We have mentioned already, that the resistance of a boundary zone is different according to the kind of locomotion concerned. The same stormy lake which is impassable for a swimmer and which a sailboat can cross only with difficulty may offer easy passage to a rugged steamer. If one wants to overcome a barrier, one usually does it not by continuing the original kind of locomotion with increased efforts but by choosing another kind of locomotion against which the barrier is weaker. The problem which the barrier sets is essentially one of finding the most suitable kind of locomotion or communication. This is true of social communication, for instance of trying to get a message to a political prisoner in spite of barriers. The use of tools also is closely connected with this question.

We shall return later to the relation between solid barriers and boundary zones which can be passed only with difficulty.

Zones of Undetermined Quality.—So far we have treated barriers and boundary zones whose qualitative characteristics were, at least to some extent, determined. However, there are rather frequently cases in which the boundary zone not only contains undetermined sectors but where the approach to a point is impossible because the intervening region cannot be determined or where it is, so to speak, psychologically "empty." Unsolved mathematical problems often offer just this kind of difficulty. One knows that it is possible to solve the problem, but one cannot see any approach to the solution. We can say about the situation only that there is within the life space a region *G* which is separated from the person *P* and which corresponds to the solution of the problem (Fig. 23*a*). But in this case the intervening region *U* between *P* and *G* does not consist of empty space in the sense of a medium which can be crossed easily. It is rather a region whose quality cannot be determined sufficiently and which therefore cannot be crossed.

If one represents such a situation more exactly, then one has to say: there is for *P* the region of the mathematical task *A* which, in so far as it is a problem, can be sufficiently characterized (Fig. 23*b*). For *P* only the starting point (*sp*) within *A*, which corresponds to the way in which the question was set, is accessible. Only this region *sp* therefore is part of *P*'s space of free movement (*C*, *D*, *E*, . . .). (The boundary of this space of free movement may correspond to the line *b*.) There is further a region *G* within *A* which corresponds to the solution. Sometimes, to be sure, it is not even certain that such a region *G* exists at all. In any case no pathway is visible from *sp* to *G* because the quality of the intervening zone cannot be determined.

The situation often develops in such a way that the person succeeds in finding a region *r* (Fig. 23*c*) which is connected with *G* and which he hopes to reach more easily from *sp*. Little by little there can appear a larger group of such regions connected with *G* (*r*; *s*, *t*, *v*). At the same time one usually tries to find more regions connected with *sp* (*c*, *d*, *e*) in such a way that one can hope finally to build a bridge from the starting region *sp* to the solution *G*, *i.e.*, a series of regions the topological sum of which $sp + c + d + e + \cdots + v + t + s + G$ is a connected region. When one has to reach a certain region by moving through such an unstructured zone it often remains uncertain, as long as one has not really completed the bridge, whether the regions *s*, *t*, *v* and *c*, *d*, *e*, which are developed at first, will or will not serve as an approach to the solution.

This type of barrier is not limited to conceptual locomotion in mathematical tasks. Similar situations can often be observed in connection with bodily locomotions. For instance one may

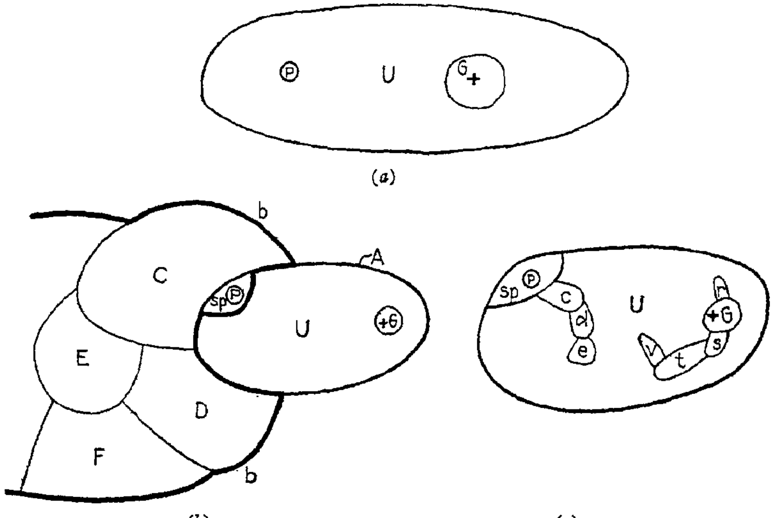


FIG. 23.—Boundary zone of undetermined quality. (*a*) A mathematical task involving an undetermined boundary zone; (*b*) situation in the beginning stage; (*c*) attempts to bridge the gap by proceeding from both ends. *A*, Region corresponding to the mathematical task; *P*, person; *G*, goal (solution of mathematical task); *U*, undetermined region between person and goal; *sp*, region corresponding to starting point; *c*, *d*, *e*, *r*, *s*, *A*, *v*, regions, determined in character, which are intended to bridge the gap between *sp* and *G*; *C*, *D*, *E*, *F*, part of *P*'s space of free movement; *b*, boundary of *P*'s space of free movement.

want to go from the railway station in a strange city to a certain house without having a map or without the possibility of asking for information. Or, one wants to find a person whose house and name one does not know. In such cases there exist the well-defined barriers of the kind we have discussed in an earlier chapter, *e.g.*, the obstacle of physical distance. But besides difficulties from such barriers the principal difficulty lies in the fact that one does not know whether any given movement brings him closer to his goal or takes him farther from it. A characteristic property of barriers of this type is that they depend directly on one's knowledge or rather on one's ignorance of the situation. According to Tolman's concept of cognitive structure one can say that the difficulty in these cases consists in the fact that the field is unstructured with reference to cognition. If, for instance, the stranger finds a map of the city the difficulty is removed.

Maze experiments with rats offer an example of the overcoming of such difficulties of locomotion on the basis of cognitive

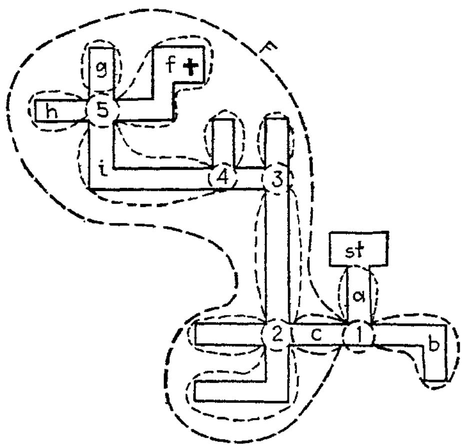


FIG. 24.—Maze learning. Connectedness or not connectedness with respect to the region containing food is the aspect according to which the field is structured. *st*, Starting point; 1, position of rat; *a*, *b*, *c*, regions adjacent to 1; *F*, region containing food; *f*, food.

structuring of the field. A rat may have found the food for the first time in a new maze. It is brought back to the starting point. It then "knows" that it is possible to get from its place to the food, but it does not yet know the path. The main task of orientation in such a maze is the following: the rat may have run from the starting point *st* (Fig. 24) to the first branching (1). It is then faced with two possibilities, to enter region *b* or to enter region *c*. The fundamental difference between *b* and *c* in this case is that region *c* is a part of a connected region *F* which contains the food $f (f < F; F > C)$, whereas region *b* (seen from station 1) is not connected with the region of the food $(F \cdot b = 0)$. The rat "knows" the path as soon as it is able to decide at each branching (2, 3, 4, 5) which of the adjacent regions "leads to" the food and which does not. In other words, the rat is able to find its way as soon as it knows the topological relationships of the regions well enough to make the correct decision at each critical point. The analogy to the mathematical task is obvious. In such maze experiments the first structuring into an articulated series of regions often occurs near the food (regions *g*, *h*, 5, *i*). In these cases the rat learns the maze from the goal backward.

If one puts a rat into the maze without food, he gets a chance to "orient" himself, which means that what is first an unstructured field becomes structured. Insofar as this process of structurization tends to be complete, the animal will know at any point the relation to the adjacent regions and perhaps also to the more distant ones.

In the case in which the animal learns from the beginning to go after the food, there, too, is a process of orientation and structurization but the structurization is a very special one with a "start" and "end." In the non-reward orientation case the structure will be much more variable and will permit of more aspects (*Auffassungen*).

What happens at the moment one puts food in the maze after giving the animal the orientation period is, to my mind, so far as cognitive processes are concerned, this: The field undergoes a restructuring so that the one aspect which enclosed the start-end relation will become dominant. One would assume, also, the following conclusions: First, after an optimum time has been given to the animal for the orientation period, an increase in the orientation time should be of no further help to the "latent learning" (85). Second, there should be cases in which the second learning should occur through one repetition. I assume thereby that the restructuring of an already structured field can be done with rats by one sudden act. This statement has certainly some limitations and will not hold for very complicated mazes or for unintelligent rats. Third, it may be possible to create mazes so that the "natural" aspect resulting from the first structuring would be of a type which would be difficult to restructure quickly. I don't know how difficult this restructuring could be made, but I think there should be appreciable differences for different set-ups.

In locomotions in the quasi-social field also a person may come to regions which he cannot cross because they do not have sufficient cognitive structuring. It is only in rare cases that the path which leads to the social goal is clearly known in advance. For instance the occupational goals of a young man are often of such a kind that the region between his present position and the goal is not at all, or only vaguely, structured with reference to cognition.

The difficulties which result from the absence of cognitive determination of the boundary zone are very common. They are essentially different from other types of barriers and from everything that one has been accustomed to think of as a barrier if one is guided by the idea of a physical barrier. But these zones actually hinder locomotion and must therefore be called barriers. They are comparable to barriers of the highest degree of solidity in so far as locomotions through such zones usually are impossible as long as their cognitive structure is not sufficiently known. On the other hand this impossibility is not a result of solidity of the barrier or of friction within the boundary zone such as we have discussed above. The impassability of such a zone depends in a special sense on the "knowledge" of the person. The friction of a cognitively well-defined boundary zone is not removed by the fact that the person concerned knows of it. The real locomotion through the region remains difficult in spite of this knowledge. On the other hand the difficulty of locomotion which results from a cognitive indetermination of a zone is actually removed by a recognition of its properties. Therefore we find in these cases a peculiar relationship between the knowledge of a zone and the possibility or impossibility of locomotion through it.

Even with qualitatively well-characterized barriers of given solidity or given friction there is a relationship to cognitive factors. A change in the knowledge of the boundary zone, or in other words a change of its cognitive structure, may show the existence of a part of the boundary zone which can be crossed more easily than the parts which were known originally. Furthermore, a more exact knowledge of boundary zones often allows one to find a different kind of locomotion by means of which one can cross the zone without meeting great resistance.

CHAPTER XIII

THE RELATIVE POSITION OF TWO REGIONS

The concepts of psychological region, of psychological boundary, of locomotion, and of communication allow us to represent an infinite number of different structures of the psychological life space. To make these representations is a task of empirical psychology. Here it must suffice to discuss a few simple cases.

Foreign Regions

The relative position of two psychological regions is topologically especially easy to represent when one simply connected region is enclosed by another. An elementary example is the space of free movement in a prison. If we disregard the differentiation within the prison, one can say: The walls of the prison, like a Jordan curve, separate an inner, connected, limited region (the space of free movement) from an outer region (the region of "freedom"). In the construction of every prison use is made of the fundamental mathematical fact that each path from a point of the inner region to a point of the outer region must intersect the Jordan curve: aside from the dynamic property of the walls (their solidity), it is above all their topological properties, namely, their arrangement as a closed curve, which makes the escape of a prisoner impossible.

The following fact too is directly connected with the topological relationships. The cell C in which the prisoner may be located represents in itself a connected limited region which is bounded by a Jordan curve.[1] In order to gain freedom he prisoner must not only surmount the boundaries of this region but also the outer walls of the prison. How many successive

[1] As noted above we refer, for the sake of simplicity, only to the two-dimensional in this discussion.

obstacles the prisoner has to surmount in making his escape depends essentially on topological relationships, namely, on how many regions (C, R_1, R_2, R_3, . . .) lie one within the other ($C < R_1 < R_2 < \cdots$) in such a way that their boundaries (b_x) have no common parts ($b_C \cdot b_{R_1} \cdot b_{R_2} \cdots = 0$. Fig. 25*a*). If the prisoner's cell lies on the outer wall (w) of the prison

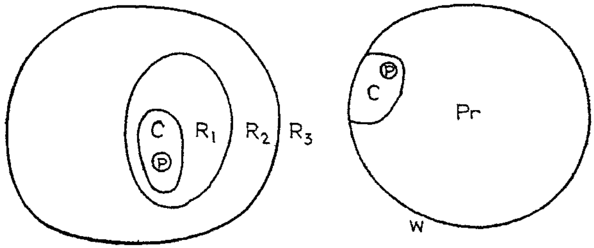


FIG. 25.—Topology of the prison. (*a*) Series of walls without common parts; (*b*) walls with common parts. *P*, person; *Pr*, prison; *w*, wall.

(Fig. 25*b*), that is, if the boundaries of the cell (b_C) and of the whole prison (b_{Pr}) have common parts ($b_{Pr} \cdot b_C \neq 0$), the prisoner would have to cross only one boundary.

If two foreign psychological regions do not correspond to an inner and an outer region which are separated by a Jordan curve, it is psychologically important to determine whether or not they have a common boundary (Figs. 10 and 15). We have already explained how this can be determined (see p. 94).

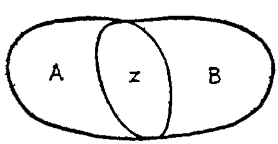


FIG. 26.—Boundary zone and the overlapping of regions. *z*, common part of *A* and *B*.

OVERLAPPING REGIONS; THE RELATIVE WEIGHT OF SITUATIONS

The cases in which two regions overlap, in part or wholly, play an important role in psychology. One can often consider the boundary zone z between two regions A and B as an area in which the two regions A and B overlap (Fig. 26). This is true for instance of boundary regions between two occupations or two branches of science. The boundary zone becomes thereby an intersection of the regions A and B both of which are defined as including z. ($z = A \cdot B$.)

A psychologically important application of the concept of overlapping is the overlapping of two situations. A child may be eating and at the same time listening to the song of a bird. The listening can be the major and the eating the minor activity, or the reverse. Between the two extremes many transitions are possible. Such cases in which one is involved to different degrees in two different activities are of common occurrence. But they offer considerable difficulty for description as well as for treatment of their dynamic facts. One can meet some of these difficulties in the following way.

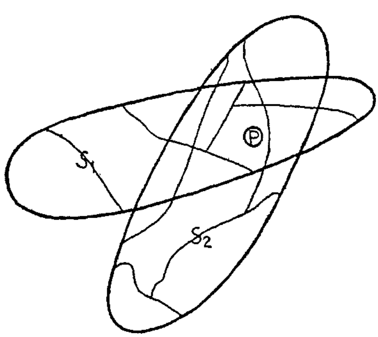


FIG. 27.—Overlapping situations. The person P is in two different situations S_1 and S_2 at the same time.

One can say in such cases that the person P is in two regions at the same time. Each of these regions is usually well structured and has the character of a situation. One can therefore speak of an overlapping of two situations. These situations (S_1 and S_2) are not foreign regions, but have a common intersection. This is proved by the fact that the person P is at the same time in both regions ($S_1 > P; S_2 > P$; therefore $S_1 \cdot S_2 \neq 0$) (Fig. 27). The psychological structure and content of the two partly or wholly overlapping situations can be very different.

If two or more situations overlap in such a way, each situation possesses at a particular moment a certain "relative weight" (importance, potency) for the person. The change of this relative weight is a dynamically important process. The forces which result from a situation seem—other things being equal—to increase and decrease with the relative weight of this situation. The change of the relative weight of a situation is one of the principal ways of influencing other persons and is often used for pedagogical purposes (18).

It is sometimes possible without special difficulty to discriminate with sufficient exactness between several degrees of relative weight and to characterize the state of the life space by the quotient of the relative weights of two or more situations.

Difficulties in Representing the Relative Position of Two Regions

As long as one limits oneself to two regions it is easy to determine their relative position and the properties of their boundaries. But when the two regions lie within a group of other regions the problem of characterizing their relations becomes a more difficult one.

In Fig. 28 for instance one can easily enough say that the regions A and B are foreign to each other; furthermore that they do not have common boundaries. But one cannot make this statement with assurance if one includes in the concept of boundary that of boundary zone. One can consider the regions 12, 4, 14, 21, 20, 19, 16, 15, 11, 10, 6, 5 as part regions of a boundary zone b_A which surrounds A; further the regions 5, 12, 15, 11, 19, 16, 10, 17, 9, 7, 2, 6, can be considered as parts of a boundary zone b_B around B. A and B would then be two regions whose boundary zones have common parts ($b_A \cdot b_B = 5 + 12 + 15 + 11 + 6 + 10 + 19 + 16$). On the other hand, one could treat only the region $4 + 14 + 21 + 20 + 15 + 12$ as the boundary zone of A and consider the topological sum of the regions 2, 7, 9, 17, 16, 10, 6, as the boundary zone of B. In this case the boundary zones of A and B would have no common part.

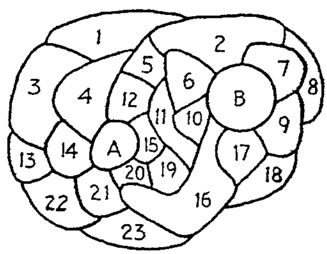


Fig. 28.—Different possibilities of viewing the boundary zones of A and B. (For example, the region $4 + 12 + 15 + 20 + 21 + 14$ might be considered as the boundary zone b_A of A; and the region $6 + 2 + 7 + 9 + 17 + 16 + 10$ as the boundary zone b_B of B; in this case the two boundary zones would have no common part; $b_A \cdot b_B = 0$. If one considers, however, the more inclusive region $1 + 3 + 4 + 5 + 6 + 10 + 11 + 12 + 15 + 19 + 20 + 21 + 22 + 13 + 14$ as b_A and region $11 + 6 + 2 + 7 + 9 + 17 + 16 + 10$ as b_B, then $b_A \cdot b_B = 6 + 10 + 11$.)

Topologically there is no reason to consider the indicated regions as a boundary zone between A and B. One could for instance instead think of the topological sum of the regions 16 and 20 as an arm which brings B into communication with A.

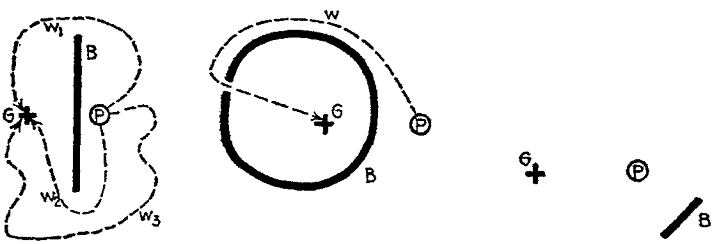
Obviously the psychological justification of these different interpretations depends on the dynamic character of the regions concerned. But the very fact that there are so many possibilities of interpretation occasionally leads to confusion in psychological research and it often requires considerable experience to know where one is dealing with intrinsic differences and where the difference is only one of expression. It seems to me therefore advisable to take up a few examples in order to discuss these difficulties. In doing so we shall only use concepts which we already have explained. But we shall employ them, as in practical experimental work, according to the requirements of the concrete problem, not in systematic order. We shall introduce these discussions in the form of a problem. In this way the relations between the different topological concepts and between the topological and dynamic concepts will become clearer.

The Two Principal Possibilities for the Representation of the Inaccessibility of a Point.—Let us represent the following situation: "A person has a certain goal, but at the moment it is difficult or impossible for him to reach it." (Vector psychology has to discuss the forces which may be involved in such a case (54, p. 253). Here we shall consider only the topological aspects of the problem.)

The cases in which an obstacle makes it difficult to reach a goal are frequent, and of many different kinds. The goal may be a certain job and the obstacle may be the fact that there is a second applicant who has more influential connections. In other cases the applicant's own incompetency, the fact that he does not yet hold certain degrees, or that he is disqualified on account of his citizenship or religion may constitute the obstacle. The goal may be going to a dance, marriage, a business transaction, or picking a flower. The obstacle may be a prohibition which rests on a law or perhaps on the authority of a person, or it may be social convention by which the person feels himself bound.

However different the cases are in detail they have at least the one fact in common, that two separate points or regions, the person *P* and the goal *G*, are distinguishable and that a barrier *B* makes locomotion from *P* to *G* difficult or impossible. As the most simple example, we can think of the case in which a physical barrier blocks the approach to the goal.

One could attempt to represent such a situation by the diagram of Figure 29*a*. A line *B* representing the barrier lies between *P* and *G*. But such a representation does not express the fundamental fact that there is no usable path between *P* and *G*. The representation offered in Fig. 29*a* leaves open the



(*a*) (*b*) (*c*)

FIG. 29.—Topologically inadequate representations of the inaccessibility of a goal. The representations (*a*), (*b*), and (*c*) are not different topologically. *P*, person; *G*, goal; *B*, barrier; w, w_1, w_2, w_3, paths.

possibility of many such paths from *P* to *G*, for instance w_1, w_2, w_3. From a topological point of view such a representation means that *P* and *G* are points of one connected region (see p. 88).

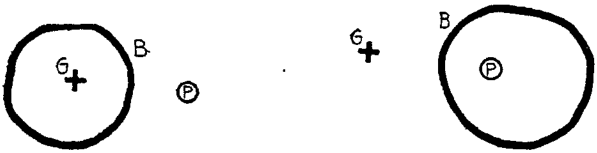
This fact is in no way changed if one gives the barrier the form of that in Fig. 29*b* where *G* is "almost entirely" enclosed by *B*. In this case too *P* and *G* lie in one connected region: there is a path w from *P* to *G* which does not cross *B*.

One has to be clear about the fact that topologically the representation in Fig. 29*b* is in no way better than that in Fig. 29*a*. Both representations are topologically equivalent even to Fig. 29*c* where *B* does not lie "between" *P* and *G* in the sense of ordinary geometry.

If one wants to represent the "unattainability" of *G* in a way which is topologically adequate, one has to be sure that *P* and *G* do not belong to one connected region. This means that one must represent the barrier as a Jordan curve which divides the whole field in such a way that P and G belong to two different regions.

We can do this in two and only two ways: either the goal G lies in the inner region I and the person P in the outer region O ($G < I$; $P < O$) (Fig. 30*a*) or the person lies in the inner region and the goal in the outer (Fig. 31*a*) ($G < O$; $P < I$).

The psychological differences between these two representations become clear when one characterizes more closely the different regions of the life space which belong to the inner and the outer field. If the person is in the outer region then relatively few regions (*a*, *b*, *c*, *G*, Fig. 30*b*) are unattainable to him. In the remaining space he can move about freely. If, on the



FIGS. 30*a* and 31*a*.—The two fundamental possibilities of representing inaccessibility: (30*a*) G lies in the inner region, P in the outer region; (31*a*) P lies in the inner, G in the outer region. *B*, barrier represented as Jordan curve; *G*, goal; *P*, person.

other hand, the person is within the barrier (Fig. 31*b*), then the space of free movement is limited to a narrow region and everything else (*a*, *b*, *c*, . . . , *g*) is unattainable. In this case the situation therefore has to a much higher degree the character of a "restraining situation" like the situation of the prison.

As an example one can use the difference between a situation in which a command is supported by means of promise of reward and a situation in which the command is supported by threat of punishment. In order to make a threat of punishment effective one has to create a restraining situation, even when the command refers to a definite task. The space of free movement of the person has to be limited to a sufficiently small region. Otherwise the person will escape at the sides (52, pp. 96*ff*.). In the case of reward the space of free movement of the person can remain unlimited. Only the access to the region of the reward is limited; *i.e.*, one cannot get the reward without first passing the region of the task. A restraining situation, or in other words, the presence of an outer barrier which makes the situation inescapable plays an important role for certain emotional processes, for instance anger (20) and

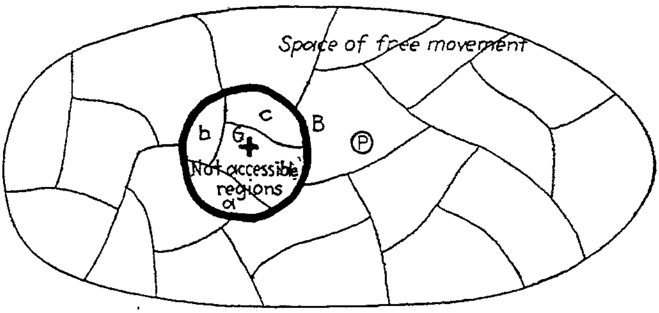


FIG. 30*b*.—Elaboration of Fig. 30*a*. Region $a + b + c + G$ is inaccessible for P

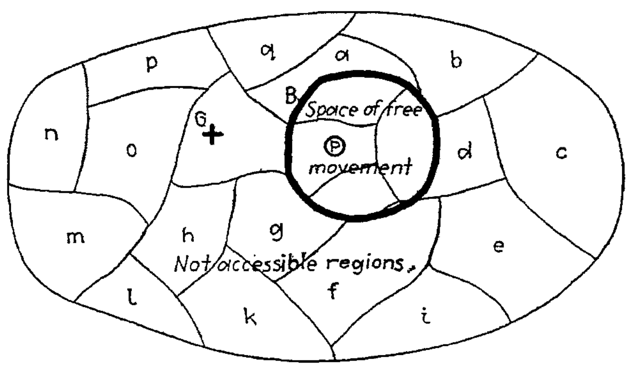


FIG. 31*b*.—Elaboration of Fig. 31*a*. Region $a + b + c + \cdots + p + q + G$ is inaccessible for P.

despair (52, p. 195; 19). In regard to the resulting forces too there are important differences between the cases in which the person is within the Jordan curve and those in which he is outside of it.

There are, in a certain sense, transitions between the two cases. Their dynamic difference rests in part on the different extent of the space of free movement. If the person is within the barrier, the space of free movement can be enlarged by including further regions in the inner field, so that the character of a restraining situation gradually becomes weaker (52, p. 128). On the other hand, if the person is outside the barrier, the enlargement of the unattainable regions within the barrier, or the appearance of further islands of the unattainable in the life space can more and more limit the space of free movement.

The size of a space of free movement is not a topological concept. Topologically one can say only what part regions belong to it. Nevertheless one can determine in this way an extension over new regions or a restriction of the space of free movement. A limited space of movement sometimes seems to grow psychologically when the region differentiates itself into a number of subregions. Even an objectively small region can, in this way, have the significance of a relatively wide field for the person concerned.

Occasionally outer barriers have at the same time the character of a protection against influences from the outside. For the gangster the prison may serve as a not unwelcome protection against attack by his rivals. Often a prisoner of many years feels unprotected against the dangers of the outer world as soon as he leaves the prison. It is then difficult to persuade him to leave his home or even to expose himself to the gaze of other people from whom he was safe while he was in prison.

In our graphic representation, the degree of solidity of a barrier is generally designated by the thickness of the line.

Topological and Dynamical Aspects of the Representation of Limitations.—One can raise the question whether it is possible to represent by topological means the fact that a goal is attainable but only with difficulty. It may seem possible to express the difficulty by leaving only a small gap in the Jordan curve and making it the smaller the greater the degree of difficulty. However one must not forget that there are no size differences in topology. As we have mentioned there is no difference between Figs. 29*b* and 29*c*. The degrees of difficulty between the impassable barrier on the one hand and the boundary which offers no resistance on the other hand cannot be characterized topologically but only dynamically. When the goal is attainable, but more or less difficult to reach, one still has to represent the barrier as a closed curve or a ring-like boundary zone. Only in this way can we express topologically the fact that there is a barrier "between" *P* and *G*. To the different degrees of difficulty correspond dynamic differences of the boundaries, for instance different degrees of solidity.

Discrete Paths and Their Totality.—One could try to represent the inaccessibility of a goal in the following way,

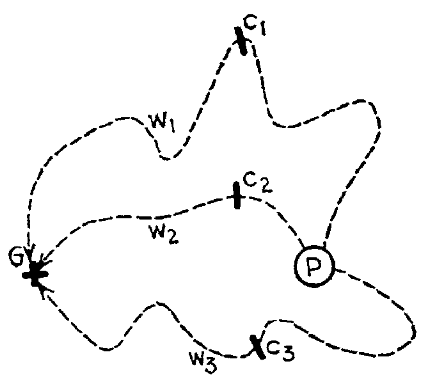


FIG. 32.—Attempt to represent inaccessibility by discrete blocked paths. *G*, goal; *P*, person; w_1, w_2, w_3, paths; c_1, c_2, c_3, blocked points.

without using a Jordan curve. One could start with certain paths w_1, w_2, w_3 between person *P* and goal *G* and could represent the impassability of the different paths by cuts c_1, c_2, c_3, which signify the impassable points in each of these paths (Fig. 32), restricting the representation of the life space to a one-dimensional space (see p. 193).

By such a representation the obstacle assumes the character of an unconnected set of discrete points. This may be adequate in a case in which we are dealing with a definite number of separate paths to the goal, as in a maze experiment. In other cases it is not correct, for (1) such a representation implies the assumption that it is impossible to go "along" the obstacle (and this doubtless is possible at times), and (2) it does not show that there are other paths open between *P* and *G* besides those which are especially indicated. The Jordan curve takes into account the totality of possible paths, but the representation of Fig. 32 does not do so.

As we have mentioned before it is very important in representing a situation to take this totality of all possible events into account. If the representation of a psychological life space is to be more than an illustration without compulsory consequences, the coordinations between psychological and mathematical facts have to be strictly maintained. Each representation in which P and G belong to a connected region, as in Fig. 32 (considered as a more-than-one-dimensional space) would show positively that there are still further paths between P and G. The representation would therefore be false in an important point.

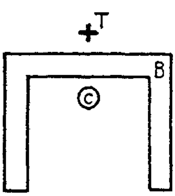


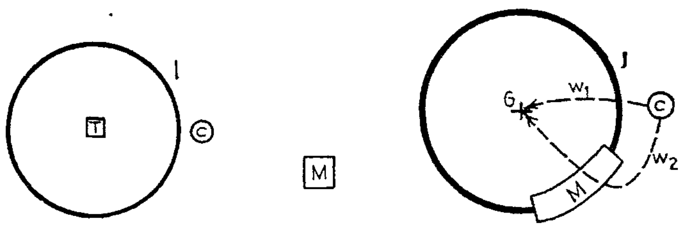
FIG. 33.—Detour problem (physical situation). *C*, child; *B*, u-shaped bench; *T*, toy.

Figure 32 is not sufficient even for the representation of the paths of a maze. The fact that there are no other possible paths is not expressed. One must represent a maze topologically as a branching of regions with barriers on both sides to prevent escape (Fig. 24). Such a representation would be correct even for an elevated maze where the paths are not enclosed by physical walls but where the rat is unable to leave the paths.

Homogeneous and Differentiated Barriers; Approach and Withdrawal.—A one-year-old child C stands behind a U-shaped bench B (Fig. 33). He wants to get the toy T on the other side of the bench but has not yet a sufficiently broad survey of the situation to carry out the necessary detour. In this case also there is a barrier for bodily locomotions which makes it impossible for C to reach T. Therefore according to our definitions C and T, as far as this locomotion is concerned, do not belong to a connected region. The barrier has to be represented in this case also as a Jordan curve (Fig. 30*a* or 31*a*) in spite of the fact that it is not closed physically. Vector psychology has to take up in detail the conceptually difficult problem of detour (see 54; 60).

We already have mentioned that the single parts of a barrier can correspond to different degrees of difficulty. We shall now discuss briefly an example, taken from our films, of a barrier which is physically homogeneous, psychologically inhomogeneous.

A toy T stands within a circular iron barrier I. A one-and one-half-year-old child C who is outside of the barrier wants to get the toy (Fig. 34*a* is a diagram of the physical relationships). Besides, the mother M is in the room. After a series of futile attempts to climb over the barrier the child runs to the mother for help. This turning toward the mother need not have the



(*a*) (*b*)

FIG. 34.—A young child wishes to reach a toy which lies inside a circular barrier. (*a*) Physical situation; (*b*) psychological situation. C, Child; T, toy; I, barrier; M, mother; G, goal; w_1, w_2, paths.

character of a turning away from the toy. But it can have the meaning of an "indirect" turning toward the toy (54, p. 253). The child may have realized suddenly that not only the iron barrier but also the mother stands between him and the goal. In this case there occurred a restructuring of the psychological situation which is indicated in Fig. 34*b*: while the barrier between C and the goal G at first had to be characterized as relatively homogeneous it consists now of at least two parts (sectors) one of which corresponds to the iron barrier I, the other to the mother M. Path w_1 corresponds to reaching the toy by crossing the physical barrier, path w_2 to getting the toy through the help of the mother.

Therefore even when we are dealing with quasi-physical fields the representation of the psychological field will have to follow exactly the relations of connectedness which are defined by the psychological functions.

Barriers and Adits.—A person is given the following problem $\frac{8 \times 3 \times 74}{2 \times 3}$. He begins by canceling the two into the eight and the three into the three. Let us represent the moment before he performs these operations. One again can start from the fact that there is a goal *G*, namely, the solution of the problem, and that the person *P* is separated from this goal by a barrier *B* which is not very strong (Fig. 30*a*).

However, if one looks more closely it may seem doubtful whether it is admissible to speak here of a "barrier." The development of the whole situation may be as follows: the person finds himself faced with a problem. The solution of the problem *Pr* is then a region outside of which *P* is located (Fig. 35*a*). This region is not an entirely homogeneous field but shows from the beginning or very soon a certain structure. Although the goal *G*, the solution, is not yet fully evident the path to the goal becomes clear. The person realizes that one must first cancel two into eight and three into three and then multiply four by seventy-four. The path to the goal is therefore characterized as a series of operations, namely, division D_1, division D_2, multiplication *M* (Fig. 35*b*).

We meet here again the fact which we have mentioned before (see p. 107), namely, that the concepts of action and of path shift between a one-dimensional locomotion and a more-dimensional region. The single steps of locomotion correspond to part regions of the task. The performance of the operations would correspond to a locomotion w_1 of *P* through these regions to *G*.

However, the representation in Fig. 35*b* is not yet satisfactory if one considers the necessity of taking into account all the consequences of the representation. Figure 35*b* leaves open the possibility that *P* reaches the goal *G* directly by way of the path w_2 without crossing regions D_1, D_2, and *M*. This would imply that *P* can arrive at the solution without really carrying out the operations. Such a process is psychologically not without meaning. The person may for instance know the result because he has made the same operations before, or because somebody has told him the answer. If, however, as we shall assume in our case, this is not true, the representation in Fig. 35*b* is not sufficient.

There are two different ways of meeting this inadequacy of representation. One can (1) use the concept of dynamic solidity of a boundary, or (2) use purely topological means.

Ad 1: Since it is a question of excluding certain locomotions, which the representation, used so far, still allows, one might represent as passable only those boundaries of the regions concerned which correspond to the path w_1. We can represent the fact that there is only one possible path to G by giving the character of an impassable barrier B to the outer boundary of the whole region Pr (Fig. 35*c*) with the exception of the boundary of D_1. Thereby one succeeds in showing that person P can reach goal G only by passing regions D_1, D_2, and M.

Ad 2: If one does not use impassable boundaries, one can limit the possibilities to this one kind of pathway by surrounding the region of the goal G by a series of concentric ring-like regions which correspond to the operations M, D_2, and D_1 (Fig. 35*d*). In this case again P's only passageway to G is D_1, D_2, and M.

The question arises in what respect the two representations agree and in what respect they are different. The most important topological difference between the two representations consists in the fact that G has in the second case (Fig. 35*d*) only one adjoining region, namely M, whereas in the first case (Fig. 35*c*) there are parts of the boundary of G which are not at the same time parts of the boundary of M. Similarly region M has in Fig. 35*d* only regions G and D_2 as neighbors; in Fig. 35*c* on the other hand there is still a further region. This holds also for D_2 and D_1. If one represents the topology of the situation by Fig. 35*b*, one can achieve limitation to one approach only by representing part of the boundary between the part regions of the problem and the surrounding regions as an impassable barrier. Figure 35*d* does not have to use the concept of the solidity of a barrier because in this representation there are no boundaries which make it possible to enter "from the side." The two representations are therefore actually somewhat different, and it is a question which is the correct one.

One could object to the representation in Fig. 35*c* on the grounds that in reality one cannot observe a solid barrier B between the regions G, M, D_2 and a general outside region. It is not such a barrier but the inner logic of the mathematical problem which makes the paths w_2, and w_3 (Fig. 35*b*) impassable. This "logical impossibility" is doubtless represented more adequately by the topology of Fig. 35*d* than by the barrier B in Fig. 35*c*, which seems somewhat arbitrary.

The representation by Fig. 35*d* may be adequate for the case in which the person concerned thinks of the sequence of the three operations D_1, D_2, and M (canceling two into eight, three into three, and multiplying by seventy-four) as the only possible path to G. As a rule the person knows that there are also other paths to G, at least the long one of performing the several operations in the order in which the problem is stated $8 \cdot 3 \cdot 74 \div 2 \div 3$ (corresponding to the regions M_2, M_3, D_3, D_4). This means that the representation of the situation by means of a barrier (Fig. 35*c*) is correct in so far as M is not the only region which adjoins G. Besides M there is at least region D_4 as a possible neighboring region of G. Before the person decided to use path D_1, D_2, M there may have been a situation when he was wavering between it and the other possibility. If one would base the representation of such a situation on the principles underlying Fig. 35*c*, it would lead to Fig. 35*e*: two possible adits (Ad_1, and Ad_2) which correspond to different operations lead from P to G. To exclude the possibility of a "direct" approach to G one would have to coordinate an outer barrier B to each of these adits.

If there are still more adits (Ad_3, Ad_4) which the person can see, one would have to include in the representation further regions and combinations of regions which are connected with G (Fig. 35*f*). It is not necessary that each of these approaches is clearly structured in advance. It can be that a path is visible as a whole, but unstructured (Ad 4).

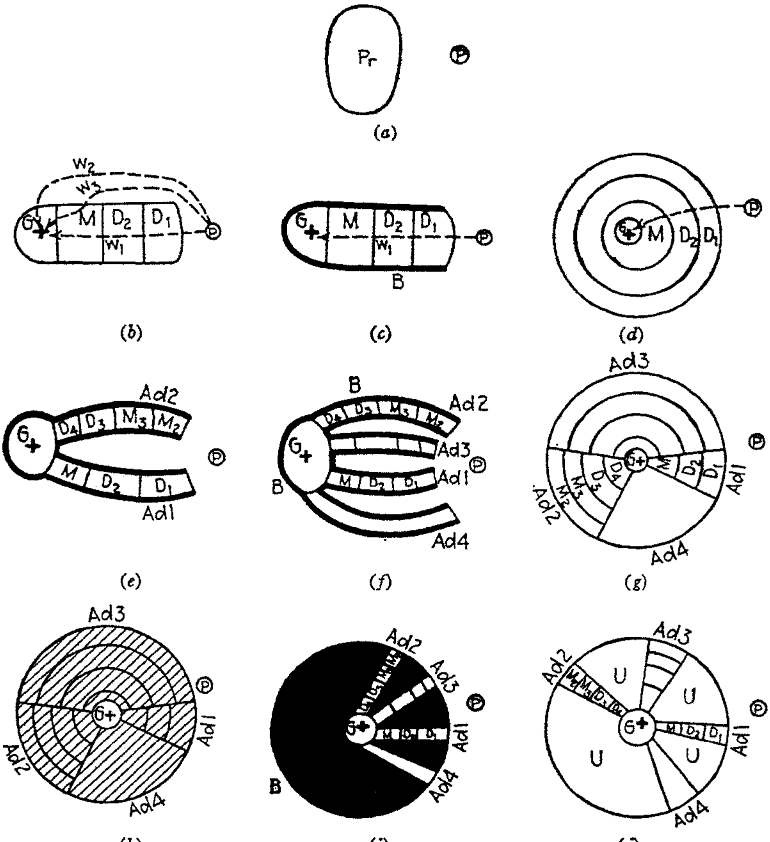


FIG. 35.—Two fundamental ways of representing a situation, when a goal can be reached only by certain approaches. This limitation of accessibility can be represented either by purely topological means or with the help of dynamical concepts. (*a*) Indicates the undifferentiated; (*b*) the differentiated situation in the beginning stage of a mathematical task without representation of the limited accessibility. The representation of this limitation by purely topological means is indicated in (*d*) and elaborated in (*g*); the representation with the help of dynamical concepts is indicated in (*c*) and elaborated in (*e*), (*f*) and (*i*). (*h*) and (*j*) show the relations between the two representations: in (*h*) the approaches are viewed as a boundary zone between *P* and *G*; (*j*) identified the impassable barrier *B* in Figs. (*c*), (*e*), (*f*), (*i*) with certain unstructured sectors *U* inserted in (*g*). *P*, person; *Pr*, mathematical problem; *G*, goal (solution of problem); *M*, M_1, M_2, different multiplications; D_1, D_2, D_3,D_4, different divisions; *B*, impassable barrier; Ad_1, Ad_2, Ad_3, Ad_4, different adits; *U*, qualitatively undetermined regions.

One can try to represent the situation which is shown in Figs. 35*e* and 35*f* in analogy to Fig. 35*d* by purely topological means. One has then to distinguish different sectors within the ring region which surrounds *G* (Fig. 35*g*). Figures 35*f* and 35*g*, which represent the same fact, in the one case by means of impassable barriers, in the other by purely topological means, are obviously more similar than the corresponding Figs. 35*c* and 35*d*. Nevertheless, the main difference is maintained: In Fig. 35*g* no further regions adjoining *G* are possible except the adits which are represented as sectors; in Fig. 35*f* this possibility exists. Besides, there are impassable barriers in Fig. 35*f* but not in Fig. 35*g*. To be sure one cannot say that there is in Fig. 35*g* no barrier at all between *P* and *G*: *P* has to overcome the difficulties of the operation if he wants to get to *G*. Therefore, in this case also there exists a barrier; but it has the character of a boundary zone which is not impassable and which consists of the part regions of the problem itself (*cf.* p. 130). When we think of the adits as one region $Ad_1 + Ad_2 + Ad_3 + Ad_4$ (Fig. 35*h*), then we see that we have again the above mentioned (Fig. 30*a*) typical constellation in which a person *P* wants to attain a goal *G*. Considered in this way Fig. 35*g* shows a barrier between *P* and *G* which is structured in sectors and also in its depth.

In Fig. 35*f* also one can conceive the region $D_1 + D_2 + M$, as part of a boundary between *G* and *P* which has the character of a passable barrier like that of Fig. 35*h*. But besides this boundary zone there are other parts of the boundary of *G* which have the character of impassable barriers.

This becomes still clearer in the following representation: We have seen that the psychologically real barrier generally has a certain depth. We know that size differences are topologically irrelevant. From that it is clear that one does not change Fig. 35*f* topologically if one presents it in the form of Fig. 35*i*. Thereby the two ways of representation (Fig. 35*g* and 35*i*) become still more similar. The topological difference consists only in that there are in Fig. 35*i* besides the four pathways (*Ad* 1, 2, 3, 4) still further sectors within the boundary zone which are missing in Fig. 35*g*. (These sectors are indicated in Fig. 35*i* by black.) One can make the representations completely analogous if one inserts more sectors (*U*) between the four adits in Fig. 35*g* (*cf.* Fig. 35*j*). The introduction of such sectors is justified if one considers that the person is often uncertain whether there are still other adits to *G*, and what characteristics they have. Figure 35*j* is an adequate representation of the situation when *P* is not certain that there are four and only four paths to *G*. (If these inserted sectors contract to zero, one gets Fig. 35*g* again.)

What do these further sectors (*U*) of the boundary zone imply for the possibility of a locomotion from *P* to *G*? The main characteristic is that they are entirely unstructured. They can be characterized qualitatively only as "possibly existing adits of some kind." We are therefore dealing with those psychologically unqualified regions which we already have treated as a special kind of psychological boundary zone (pp. 130*ff*.). As long as it is impossible for *P* to determine their quality they cannot be used as paths to *G*. The *U* zone therefore represents an impassable barrier for *P*. In this sense Fig. 35*j* actually corresponds to Fig. 35*i*. Thereby the relationship between the two methods of representation with which we began, that of the dynamic barrier on the one hand and of the topological structure on the other hand, becomes evident.

This example may have clarified once more the previously discussed connections between a boundary and a boundary zone, between a boundary and a barrier, and between the dynamically different kinds of barrier. Furthermore it brings out the connection between boundary and path.

It is not a peculiarity of this case that the regions M, D_1, D_2 appear on the one hand as a pathway from *P* to *G*, and on the other hand as a barrier between *P* and *G*. Rather, it is a general property of every boundary that it connects and at the same time separates two regions. This is especially obvious if we are dealing not with one-dimensional boundaries but with boundary zones. The fundamental fact is that there are three regions, *A*, *B*, and *C*, and that the path leads from a point in *A* to a point in *C* by way of *B*. It depends upon one's point of view and also upon the ease with which *B* can be crossed whether one prefers to treat *B* as a boundary zone between *A* and *C* or to treat it as a pathway from *A* to *C*. In reality both points of view are always possible and have to be taken into consideration.

CHAPTER XIV

STRUCTURAL CHANGES

Differentiation, Integration, and Restructuring

A region which is at first homogeneous can become articulated into a number of part regions. Such differentiations are among the most frequent and important psychological processes. There hardly exists a psychological problem in which they are not involved in one way or another. Topologically one can think of them as a breaking up of regions into subregions which can easily be treated mathematically. Naturally there exists an unlimited number of possible variations in the kind, speed, sequence, and degree of differentiation into part regions.

These differentiations can be closely connected with cognitive processes (85, p. 440), for instance with experience or with an act of insight (46). Differentiation may result from other causes too. The development of the life space from infancy to adulthood can be characterized to a large extent as a process of differentiation (48; 23, p. 263; 81, pp. 129–137 and 162).

Perhaps not less often than a differentiation one can observe the mathematically opposite process of a dedifferentiation or integration. Such a unification can be observed for instance in certain emotional situations (20, p. 118). In these cases it is usually the effect of strong tensions. In other cases such a unification of systems, which at first are separated, is produced by intellectual processes.

Finally, we find a group of changes of the surrounding field which one cannot think of as differentiation or integration. The number of part regions of a whole may remain the same, although their relative position is changed. In such a case we shall speak of "restructuring." Very often a loss or increase of differentiation may accompany a restructuring.

Naturally, an endless number of such changes of structure is possible.[1] Their special characteristics have to be determined in each single case. Like other courses of events one will have to represent differentiations, integrations, or changes of structure as a series of situations which correspond to the different cross sections of time.

Changes of Structure and Locomotion

The change of the structure of a region has usually a different character from that of a locomotion. Nevertheless, there is a close connection between the two processes. Topologically a locomotion of a person P from a region A to a region B always implies a restructuring of the whole field: as a result of the locomotion the region that corresponds to the person P becomes part of a different region ($P < B$, instead of $P < A$).

The same is true of the locomotion of other persons or objects in the life space. Each such change of position implies a more or less important change of the structure of the environment. This is especially clear in cases in which the environment moves although the person does not actively contribute to the movement (*cf.* p. 114).

The fact that a communication is brought about between two regions by the reaching out of an "arm" (p. 102) can be thought of as a kind of transition between locomotions and other changes of structure. Such a locomotion of an "arm" can change the structure of the life space to a considerable degree although the regions involved do not themselves show a marked movement.

Changes of Magnitude and of Distance

In the case of a differentiation of a whole region, it might seem possible to speak of the formation of smaller part regions; in the case of an integration, of the formation of larger regions. This however can be done only under special conditions. As

[1] S. Fajans (23), pp. 240*ff.*, and Sliosberg (81), pp. 129–137, describe structural changes resulting from difficulties in reaching a goal and in the situation of embarrassment. K. Lewin, (52) pp. 114–170, discusses changes in a situation of reward and punishment; T. Dembo and E. Hanfmann (19) compare situations of patients in a mental hospital.

we have mentioned, the topological characteristics are independent of quantitative determinations. One cannot expect therefore to express changes of magnitude or distance by means of topological concepts.

In certain cases, however, it is possible in psychology to make statements about size or rather changes of size on the basis of topological determination. Let us consider an example: it is not possible topologically to say that region *A* in Fig. 36*a* is larger than region *B*. However if *B* is entirely included in *A*

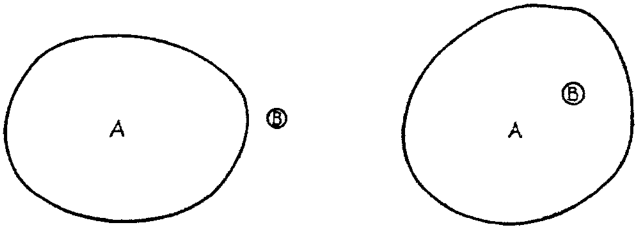


FIG. 36.—Topological conditions (*a*) unfavorable and (*b*) favorable to the comparison of size in the life space. Regions *A* and *B* may be psychologically comparable as to size if $B < A$ and $A = B + N$; $N \neq 0$.

(Fig. 36*b*) and if *A* has part regions other than *B*, we can sometimes say psychologically that *A* is larger than *B*.

In this case as well one cannot speak of size without going beyond topological concepts. Topology can only determine the relation of "being-contained-in" or of "part-whole." From the point of view of the theory of sets there is as a rule no size difference between the whole region and one of its parts: their points can be coordinated in one to one correspondence.

A simple example of the shrinking of a region is the change in the space of free movement of the child *A* in the bathtub example. After the boy *B* has established a boundary for the movements of *A* across the middle of the bathtub (Fig. 1*b*) *A*'s space of free movement is definitely smaller than it was in the situation represented in Fig. 1*a*. Parts which previously belonged to the space are excluded and no new regions are added.

Another typical example of the shrinking of the space of free movement is that which occurs for a first child when a second child is born. The first child may, for instance, have to share his room, his table, his toys with the new one. The mother can no longer give so much time to the first child and the child must be considerate of the smaller one in many ways. Each new prohibition limits the space of free movement. On the other hand the lifting of a prohibition or the acquisition of a new skill results in an extension of the space.

Such statements about the size of the whole space of free movement can be made only if, aside from the addition or loss of certain part regions, there are no significant changes in the size of other parts. After a marked reduction of the space of free movement there can appear adaptations which psychologically compensate, at least in part, for the shrinking. The remaining regions can be differentiated into subregions in such a way that it becomes doubtful whether, from a psychological point of view, the life space has become really smaller than it was before.

Nevertheless, at least at the moment in which sufficiently large part regions are taken away or added, there is a marked increase or decrease in the size of the whole space of free movement. The other part regions usually remain sufficiently unchanged at least for a short time. Therefore the velocity of the change is often of great importance for its effect on behavior.

Like size relationships psychological relationships of distance can be represented by topological means only when the regions which are coordinated to the distance *m* are entirely part of those regions which determine distance *n*. For instance, in the example of the child who wanted to carry his ball up the steps (see p. 114) one is justified in saying that the distance between goal *G* and child *C* has decreased between the first and second stages (Fig. 20*a* and *b*) and that it has increased between the second and third stages (Fig. 20*b* and *c*). In the first case there occurs no change except that certain part regions of the intervening zones drop out. In the other case new regions are added without important change in the properties of the part regions.

In dealing with dynamical problems it is often a question not of comparing any given situations with each other, but rather of determining changes of situation. Therefore one can more often make statements about size on the basis of topological concepts than one would expect in view of the rather special or specific conditions to which such statements are limited.

Dynamic Conditions of Structural Changes; Fluidity, Elasticity, Plasticity

Changes of the structure of the life space often have important dynamic consequences and depend directly on dynamic factors, especially on the distribution and magnitude of forces. The treatment of these questions therefore presupposes the concepts of vector psychology. At this point however we shall briefly indicate several differences in state of regions which are important for structural changes.

In discussing thing and medium (see p. 115) we have treated dynamical properties of regions which are important for locomotion. Furthermore we have attributed different degrees of solidity to the boundaries and we have spoken of boundary zones which offer different degrees of friction. This characterization was based upon the resistance which these boundaries or zones offer to locomotion. One has to ask a corresponding question in regard to the ease with which the structure of the field can be changed.

We can speak of different degrees of fluidity of the situation.

Definition: A situation is the more fluid the smaller the forces which are necessary, other conditions being equal, to produce a given change in the situation.

The greater fluidity can be a general characteristic of the situation in the sense that it is more fluid in regard to all sorts of influences. It would then have equal effect for instance in regard to the release of tension systems and in regard to changes of topological structure. Generally the fluidity of the situation is different for different kinds of influences. Resistance to locomotion can therefore be treated as a special kind of fluidity.

The degree of fluidity of a situation plays an important role in all processes and is one of the fundamental dynamic properties of a situation. Regions of greater irreality (see p. 196) generally correspond to more fluid media than regions of a higher degree of reality. Within the level of reality also different regions seem to possess very different degrees of fluidity. For pedagogical reasons one often tries to give a relatively great stability to certain regions within the life space of the child (regions which are connected with the routine of dressing, eating, or sleeping); there are other regions in which the child ought to feel really free and which one tries to keep relatively fluid. The general degree of fluidity of the environment is pedagogically very important. The degree of social stability of a group to which a person belongs is usually different at different times. An inflation, for instance, can lead to a great fluidity of the social field.

The fluidity of the environment is closely connected with the state of the person. Fatigue seems to produce an instability not only of the person but also of the psychological environment. The frequently noted affectivity of the small child when he first awakens from sleep may be directly connected with the instability of the surrounding field at this moment. In general, situations are most fluid *in statu nascendi*. There seems to exist a close connection between the cognitive uncertainty of the structure of a situation and its general degree of fluidity. The solidity of a region generally increases as it remains constant over a longer period of time. There are however exceptions. Sliosberg has shown that it is necessary to distinguish between different degrees of fluidity for regions which correspond to certain tasks and play materials (81, pp. 148–149 and 176–177). Playful actions and play situations generally have a more fluid character than serious ones. Frank (25*a*, p. 293) has shown that the level of aspiration is more easily raised in a playlike situation.

There are two ways in which one can represent different degrees of fluidity: (1) by attributing to different regions as wholes the qualitative characteristic of greater or less change-ability; (2) by ascribing to the regions boundaries of different degrees of solidity. In the latter case the average fluidity of the whole region depends on the solidity of the inner framework which is characterized by the boundaries of the part regions. The main difference between the two ways of representation is the following: in the first case the single part region (*a* or *b*) is considered as dynamically homogeneous; in the other case one distinguishes between the solidity of the border and that of the inner part of the single subregions. If the solidity of the region *a* is greater than that of the region *b* it would mean in the first representation that locomotions within the region *a* are more difficult than those within the region *b*. According to the second representation locomotions within *a* may be carried out as easily as locomotions within *b*.

The concrete situation therefore determines which of the two representations is more adequate in a given case. There is no doubt that a weakening or loss of the boundaries between the different regions of the environment can lead to a marked fluidity of the whole field. This can be observed in the social field in revolutionary times when the barriers between groups or barriers established by prohibitions break down; or when a child who has been brought up in strict obedience is suddenly placed in a field in which barriers of prohibition are not clearly evident. The individual differences in such cases show that in addition to the solidity of special boundaries one always has to deal with the general stability of the particular life space. Therefore one must always consider the characteristics both of the boundaries and of the regions themselves or, as we shall call it, their "material."

The definition of fluidity which we have given leaves open the question of "elasticity."

Definition: By elasticity we understand the tendency of a changed region to return to its original state. It is obvious that the degree of elasticity can be different in regions of the same fluidity.

Both elasticity and fluidity are involved in "plasticity." By plasticity one should understand the ease with which a relatively lasting and stable change can be made in the structure of a region. A too low as well as a too high degree of fluidity is unfavorable for plasticity. As a matter of fact not only very rigid persons but also certain psychopathic types which are of especially high fluidity are difficult to mold; also a too high degree of elasticity of a region can be disadvantageous to its plasticity. The plasticity of the environment depends in part on the state of the person. Katz (41, p. 127) mentioned the relation between plasticity and need. Certainly the plasticity of the life space changes in the course of the development of the person (see p. 190).

CHAPTER XV

THE LIFE SPACE AS FINITELY STRUCTURED SPACE

We shall now conclude our considerations of the topology of the environment. But before we discuss the topology of the person we must point out a fundamental fact which concerns the whole life space.

In treating mathematical spaces it is assumed that space is infinitely divisible. It is possible to break up each region[1] into part regions and therefore to distinguish part regions of part regions ad infinitum. This property of space is also presupposed by topology. One has to ask whether the psychological life space has these characteristics. In the course of our considerations we have twice met facts which are related to this problem—namely, in discussing how accurately one can determine points in the life space (see p. 111) and in treating the psychologically unqualified regions (see p. 130).

As we have seen one can determine the relative position of a point in the life space only by reference to the region in which it lies. The accuracy of this determination depends therefore on how far one can proceed in dividing regions into subregions. Few if any psychological regions can be divided into smaller subparts ad infinitum. In the example of the child who had to eat something that he did not want (see p. 97) one can distinguish the regions: putting the hand on the table; taking the spoon; bringing it part way to the mouth; etc. But it is not admissible to distinguish for instance within the region "taking the spoon" as special regions "moving the hand forward a hundredth of a millimeter," "moving the hand forward a second hundredth of a millimeter," etc. In other words it is usually possible to divide psychological whole regions into part regions but this can not be carried on ad

[1] More exactly: each more-than-o-dimensional region.

infinitum. An analysis which attempts to do this is not more accurate; it is psychologically wrong.

How far one may proceed with the division depends on the case at hand. If a child likes to eat a certain kind of food and conveying a bite to his mouth is one single process, then the distinction between the regions "bringing half way to the mouth" and "bringing to the mouth" may be an inadmissible analysis of psychologically unstructured unities. The extent to which regions can be divided therefore varies. It depends on the momentary condition of the region concerned. (We have given examples of changes in degree of structuration in discussing the boundary zone (see p. 121), further in the chapter on thing and medium.) But it always has a certain value for a certain region of the life space at a given moment; in other words it has an objective character.

A consideration of the unqualified zones which can act as barriers leads to the same result. These zones are usually in themselves unstructured (see examples, p. 131). With increasing orientation they may gradually differentiate themselves; but at a given moment there are always certain part regions within which it is psychologically impossible to distinguish further parts.

We shall call a region which cannot be divided into distinguishable part regions "unstructured"; a region whose part regions can be divided indefinitely into further part regions "infinitely structured"; a region in which division into part regions is possible, but cannot be carried out indefinitely, "finitely structured."[1] In this sense we shall speak of unstructured, of finitely and infinitely structured spaces. The psychological life space is thus a finitely structured space.

It is not possible to discuss the question here whether physical space is infinitely or only finitely structured. Heisenberg's principle of indeterminacy which suggests itself at this point does not imply that physical space is finitely structured.

[1] This term is less likely to lead to misunderstanding than the term "strukturiert" (structured) which I have used in "Der Richtungsbegriff in der Psychologie" (54).

For in regard to fields which are not in motion there are, according to this principle, no absolute limits for the accuracy of the determination of position. At least there is a difference in degree of structuredness between the physical space and the life space. While physical space is everywhere structured to the microscopic level the psychological life space often contains macroscopic regions which are unstructured. Besides, the limits of structuredness of the life space vary to a very high degree.

Riemann (76) mentions in his well-known treatise *Ueber die Hypothesen, welche der Geometrie zu Grunde liegen* that it is not necessary logically that spaces should be infinitely divisible. As far as I know, mathematics has not yet followed up this suggestion; it would certainly be of greatest interest for psychology. However, topology allows a representation of the psychological life space which is sufficiently adequate for most problems.

C. TOPOLOGY OF THE PERSON

CHAPTER XVI

THE PERSON AS A DIFFERENTIATED REGION IN THE LIFE SPACE

Up to this point we have represented the person as a connected and undifferentiated region or point within the life space. But such a representation can serve only as a first approximation. In reality the person is not an entirely homogeneous unity but a highly differentiated object (67*a*).

That psychology has to distinguish parts within the region which represents the person can be demonstrated mathematically by pointing out two facts. As we have seen it is not necessary that the whole person participate in the bodily locomotion. Sometimes only one hand reaches toward the goal, or only the eyes are directed at an object. It may even happen that different parts of the body are carrying out different activities at the same time. This means mathematically that one has to distinguish parts within the region which represents the person.

For psychology a second fact is still more important. As we have seen the behavior depends on the state of the environment and that of the person: $B = f(PE)$. In this equation P and E are not independent variables. The structure of the environment and the constellation of forces in it vary with the desires and needs, or in general with the state of the person. It is possible to determine in detail the dependency of certain facts in the environment (*e.g.*, the decrease of field forces, change of valences) on the state of certain needs (*e.g.*, the extent to which they are satiated). Thereby it becomes evident that a change of a certain need, for instance its satiation, does not change all needs in the same direction and to the same extent. This makes it necessary to distinguish within the person a multitude of different regions whose changes of state are to a certain extent independent of each other.

These considerations meet an objection which is sometimes raised against our representations. It is said that it suffices for the derivation of behavior to represent either environment or person. In reality however it is impossible to derive the psychological processes in the life space without including changes both of person and of environment in the representation. (All so-called physiological theories which do not contain a representation of the environment are for this reason inadequate.)

One will ask for criteria on the basis of which one can determine what is to be represented as a region of the environment and what as a region of the person. In answering this question it could be pointed out that the "self" is experienced as a region within the whole field (44, pp. 319*ff*.). This criterion is however not sufficient. We have seen that the goals and concepts which popular psychology has often attributed to the inner person as a rule have to be represented as part of the environment. From a dynamic point of view the following facts may be considered: one can treat everything as environment in which, toward which, or away from which the person as a whole can perform locomotion.

One will have to treat the question whether a psychological region belongs to person or to environment with the same topological methods by means of which one determines other positions in the life space. These determinations depend on the concrete facts of the individual case. Therefore for different life spaces there may be considerable differences in the structure and boundaries of the person. However, the agreement is great enough to allow us to make several general statements.

CHAPTER XVII

FUNDAMENTAL CONCEPTS AND COORDINATING DEFINITIONS FOR THE REPRESENTATION OF THE PERSON

COORDINATING DEFINITIONS FOR ENVIRONMENT AND FOR PERSON

The mathematical concepts for the representation of the psychological person are the same as those for the environment. The coordinating definitions also can remain unchanged. Nevertheless, the determination of the structure of the person has a different character from that of the environment. This results from the fact that in constructing the person we cannot use locomotion as a basic dynamic operation. For, from a psychological point of view the person himself cannot in general be considered as a medium within which an object carries out locomotions from one part region to another. In determining boundaries and connections between part regions within the person one must rather consider a general dynamic relationship, namely, the "degree of dynamic dependency" of one region upon another.

Dynamic Dependency.—It can easily be shown that the different parts of the person differ in the degree to which they are related to each other. It may be that the fulfillment of a wish changes the whole person, for instance his behavior in business as well as his behavior toward his family and toward his friends. In other cases most of the personal regions may remain almost entirely uninfluenced by the fulfillment of a wish. There are great differences in the extent of the personal regions which are essentially influenced by events in one region.

The dynamic interdependency of two regions implies that the state of the one is influenced by the state of the other. It coincides therefore with the concept of dynamic communication which we have already defined (see p. 126). It is characteristic for the determination of the topology of the person that it has to be based almost exclusively on communications and degrees of communication.

We begin by defining the concept of region of influence by the following coordinating definition.

Definition: a and b are parts of a connected[1] region (region of influence) if a change of state of a results in a change of state of b.

This definition obviously does not allow the determination of the boundary of the part regions (a, b, . . .) and their relative positions. But it does allow the determination of the including whole region and its boundaries. Let a, b, c, d, e, f, g, . . . be regions within the person. One can then determine which other part regions belong to the same whole region as a if one finds out whether or not a certain change of a changes the state of b, c, d, . . . If, for instance, the state of b, d, e would change with that of a, but the state of c, f, g would not change, then a, b, d, e would be part of one region A ($A > a$; $A > b$; $A > e$; $A > d$); c, f, g on the other hand would be parts of regions foreign to A ($A \cdot c = 0$; $A \cdot f = 0$; $A \cdot g = 0$). A change of c may show further that c and f are parts of the same region B to which g does not belong ($B > c$; $B > f$; $B \cdot g = 0$). A constellation which could correspond to Fig. 37 would thereby be ascertained. (The fact that regions A, B, C are foreign to each other does not tell whether or not they have common boundaries.)

It is clear that these determinations of regions would give different results if one started with changes of different kinds. We met a similar dependency with all determinations of region. Furthermore, we arrive at different wholes according to the part regions with which we begin. This is only an expression of the fact that the degree of communication between two

[1] In the long run it may prove to be more fruitful to use the concept of "region" in this definition instead of that of "connected region" and to determine the connection by an investigation of the "paths of influence" (see p. 172).

regions *a* and *b* can be different in the direction *a* to *b* from that in the direction *b* to *a*.

The fact that the wholes are different for different kinds of changes of state makes it possible to proceed step by step in the determination of the topological structure of regions: one can produce a second change of state of *a* which is different from the first. Then it is usually a different group of regions which is involved in the change: it may contain the region *d* (Fig. 38); further the regions *c*, *h*, *l*, which were not affected the first

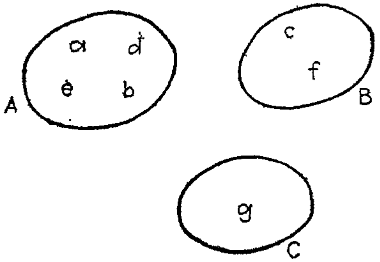


FIG. 37.—Determining "regions of influence" and their limits by means of the dynamical interdependence of their parts. For instance, a change of the state of *a* influences *b*, *d*, *e* but not *c*, *f*, *g*. $A > a + b + d + e$; $B > c + f$; $C > g$; $A \cdot B \cdot C = 0$.

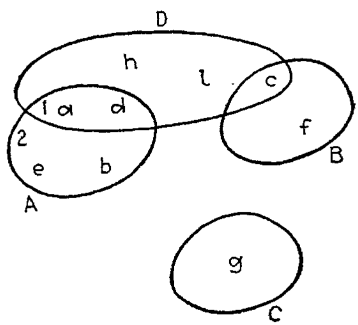


FIG. 38.—Regions determined by a kind of change of state, which is different from the kind determining Fig. 37. In this case a change in the state of *a* does not influence *e*, *b*, but *d*, *h*, *l*, *c*. $D > a + d + h + l + c$; $D \cdot b = 0$; $D \cdot e = 0$.

time. Regions *e* and *b* may show no considerable change. The region *D* which is thus defined by the new change of state has a part in common with *A* and *B*. Thereby a certain relation between the regions *A* and *B* is determined. Further it is now possible to distinguish within the region *A* two subparts (1 and 2) of which the one contains *a* and *d*, the other *e* and *b*. For the second kind of influence the communication between the two subparts (1 and 2) is weaker than between *a* and *d*. This determines a psychological boundary within *A* which has the character of a dynamic wall in the sense defined (see p. 126). Similarly region *B* is divided into two parts, region *D* into three parts by the overlapping of *A*, *B*, and *D*.

The following procedure is especially valuable. One can vary the degree of the change of state and keep both the nature of the change and the region *a* in which it starts constant. On the whole one can assume that the stronger the change of state of *a* the more inclusive the affected region. In this way one comes to a series of regions which lie entirely within each other (*M*, *N*, *O*; Fig. 39) and which correspond to the different degrees of intensity of the change in the initial region *a*.

This principle can be applied frequently, although not without exception. It can happen that the kind of process suddenly changes if a stimulus is increased beyond a certain point. Such an increase also can have the effect that the regions become dynamically closed against each other. Therefore it is possible that under certain circumstances a smaller region of influence corresponds to a stronger change of the initial region. Whether one is dealing with such an exception can be determined by means of the principles which we have given. However, the assumption holds for most processes and initial regions as long as the influence is not too greatly increased. Extremely strong influences regardless of their kind usually affect the state of the whole person.

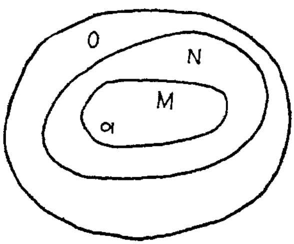


FIG. 39.—Different degrees of change in the state of *a* usually affect different areas. *M*, region influenced by a slight change of *a*; *N*, region affected by a greater change of *a*; *O*, region affected by a still greater change in the state of *a*. $O > N > M > a$.

These methods by which one can determine topological relations on the basis of dynamic dependence make use of the concept of "being-contained-in" or "part-whole." Besides one can make determinations whose form suggests the concept of locomotion and of path. Two regions usually can influence each other in different ways within an area of communicating regions. (Regions *A* and *B* in Fig. 28, for example, could communicate by way of region 20 + 16; or through 15 + 11 + 6; or through 14 + 3 + 1 + 2, etc.) The way in which a certain influence is mediated between two regions depends on the nature of the regions concerned and the special structure and properties of the possible intermediate regions. One often can distinguish between different "paths of influence" and it is possible sometimes to determine the one which is actually used. We have mentioned for instance that a person can be influenced by "looking at" only when his eyes can be caught. One can say that many influences by which the environment affects the inner-personal regions occur by way of perception. Whether or not one can induce a person to perform a certain action depends not only on which inner regions are touched but also on the sequence in which they are touched.

It may be clear without further explanation that one can use determinations of such paths of influence in making topological statements about the relative position of regions and their boundaries or boundary zones. We are then dealing not with movements of regions but with "movements" of changes of state. In these cases impassable boundary zones correspond dynamically to walls rather than to barriers (see p. 126).

Boundaries and Boundary Zones.—The representation of the inner structure of the person, in so far as it is based on dynamic dependencies, uses therefore as constructive elements regions which correspond to dynamic unities of the highest degree. They are regions whose parts are so closely connected that each change of one part results in a change of all other parts. These regions are therefore dynamic unities or gestalten in the sense in which Köhler (43) uses the word.

The unity of these regions always is determined in relation to a special kind of change. In regard to other kinds of change the parts of these regions may be dynamically separated. But this does not affect their dependency in relation to the first kind of change. The unity of the regions is therefore, in spite of its relativity, objective in character.

The dynamic dependency or independency of regions rests (1) on the qualitative properties of the regions concerned, (2) on the properties of their boundaries and boundary zones.

Ad 1: It is possible that two regions, *a* and *b*, are in immediate contact with each other and not separated by any dynamic walls. Nevertheless their qualitative properties can be such that a change in the state of *a* has no apparent influence on the state of *b*.

Ad 2: The qualitative properties of two regions could be such that an interdependency of their states would exist if they were not separated by certain boundaries or boundary zones.

Therefore the boundaries within the person, like those within the environment, rest in part on a qualitative difference between neighboring regions, in part on the properties of the boundaries and boundary zones themselves. Boundaries within the person also exhibit different degrees of dynamic penetrability; to these correspond different degrees of interdependency of regions or—if one thinks of the interdependent regions as parts of a whole region—different degrees of dynamic unity.

The mathematical concepts allow us to distinguish only connected and not-connected regions. In terms of topology there are no transitional cases. Dynamically, however, there are doubtless transitions between completely dependent and completely independent regions. We have already met an analogous difficulty in discussing the mathematical concept "boundary" and the dynamical concept "barrier" in regard to the psychological environment (see p. 144). In determining the structure of the person also we have to use certain dynamic characterizations of regions and boundary zones which go beyond the mathematical concepts.

Remarks about Strong and Weak Gestalten; Gestalten with Different Degrees of Dynamic Unity.—We are obviously dealing here with the same considerations which have been fundamental for the concept of dynamic gestalten. One might be tempted to use the concept of weak and strong gestalt in Köhler's sense to characterize the different degrees of dynamic connectedness. However, Köhler, as he has told me, feels that these terms should be used in their original sense according to which one has to speak of a strong gestalt if the change of one part of the gestalt involves changes in the form (changes of spacial distribution) of the other parts (43, p. 161). One speaks of a weak gestalt if no changes of form occur even in those cases in which the interdependency of the parts is very great. These terms therefore do not refer to different degrees of dependency. In order to express the degree of dependency of the parts of a dynamic whole we will speak of gestalten of greater or less unity. This distinction is not based upon a logical dichotomy as is the distinction between strong and weak gestalten. Instead it signifies a continuous series with the "and-sum" (Wertheimer), that is, a group of dynamically disconnected regions, at the one extreme and at the other a gestalt of the highest degree of interdependency of parts.

Dynamic Properties of Personal Regions

The determination of the topology and dynamics of the inner personal regions has on the whole a more indirect and abstract character than the determination of the structure of the environment; probably because dynamic interdependence and not locomotion is the main determinant of the topology of the person.

In so far as we consider the position of the parts of the person we shall speak of regions. When we are dealing with their state we shall speak of systems, especially if we have to deal with state of tension.

There is a great variety of influences which can be used to determine the mutual dependency and position of part regions of the person, for instance fatigue and psychological satiation. The process of satiation of a certain action depends, as Karsten (40, pp. 197–227) has shown, on the relationship of the corresponding region within the person to its neighboring regions. Conversely one can therefore determine, on the basis of observations of satiation, relationships of neighborhood and of connectedness within the person. Of the many different kinds of influences we shall discuss only the change of state of tension.

Tension.—To discuss the nature of tension in detail one has to consider vector problems. In this place we can give only a general characterization. Tension is a state of a region.

Strictly speaking one can determine only differences of tension; a difference in tension tends to produce changes in the direction of a leveling of tension. Tension is therefore a state of a region relative to that of another region and it involves certain forces at the boundaries of the region.

Experimental investigations of different kinds show that certain characteristics of the environment, especially the presence of a goal or the tendency to a locomotion, are connected with a state of tension in the person. The carrying out of a locomotion or the reaching of a goal can at the same time mean the release of a tension. The experiments show further that although to a certain extent this change involves the person as a whole, one can satisfy or leave unsatisfied different needs more or less independently. Therefore one has to coordinate to these needs states in different part regions of the person. Thus within the person we can speak of different systems whose degree of tension can change relatively independently.

Groups of Tension Systems.—To determine the way in which different tension systems are connected one can make use of their mutual dependency, applying the method that we have discussed above. A common, relatively simple relation between two tension systems is given if one is a part of another. That holds, for instance, if the one system corresponds to a subgoal of a more inclusive goal. The tension of the part system will then usually cease in case the more inclusive system loses its tension.[1]

The problem of substitute satisfaction offers another case of connection between two systems. An action *b* has a dynamic substitute value (56; 62, pp. 226*ff*.; 64, pp. 31–32) for the action *a* when the tension in the system which corresponds to *a* is released as soon as the tension system corresponding to *b* is released. That means that the two systems *a* and *b* must be sufficiently connected parts of one larger system (Fig. 40*a*); they must not be dynamically independent systems (Fig. 40*b*).

[1] Ovsiankina (68), p. 351. The part-whole relation of inner-personal regions plays an important part in papers by Schwarz (77, 78) and D. K. Adams, (1).

It has been shown that the substitute value of *b* depends upon the degree to which *a* and *b* are unified (62, pp. 243*ff.*).

There is a peculiar relation between substitute action and the use of tools or preparatory actions (56). This relation becomes understandable if one remembers that in both cases the problem of relative dependency of two systems is involved: in case of substitution the relation between two part systems s_1 and s_2 within one whole system S ($S > s_1 + s_2$), in the second case the relation between a part system s_1 to its whole $s(S > s_1)$. It is clear that these relations are not entirely different.

As Zeigarnik (92) and Bierenbaum (4, pp. 134*ff.*) have demonstrated it is possible to produce experimentally relatively

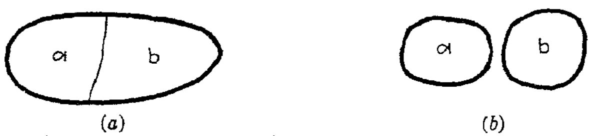


FIG. 40.—Conditions under which one activity has substitute value for another. (*a*) System *a* related to the original task, and system *b* related to the substitute task are parts of one connected system; (*b*) the systems *a* and *b* are dynamically separated.

complicated groups of systems within the person and to determine to a considerable extent the structure of these systems, especially the kind and degree of connection within the group. For instance it is possible to produce systems corresponding to a series of tasks in such a way that in one case a closely connected group and in another case (with the same tasks) relatively isolated systems arise. The position of a single tension system within or outside a group of systems also can be controlled experimentally. Bierenbaum was able to trace in detail the process of unification of systems which were originally separate.

A number of quite different investigations of satiation (40, pp. 201*ff.*), tension (92), forgetting (4, 25, 75), and substitution (62, pp. 232*ff.*), have agreed in showing that as a rule psychologically adjacent regions within the person correspond to actions or tasks which are related as to their content, although this is a principle which must be applied cautiously.

Structure of the Person

Inner-personal Regions and the Motor-perceptual Region.—If, on the basis of these considerations, we try to determine the structure of the person as a whole, we come to the following interpretation. The person is to be represented as a connected region which is separated from the environment by a Jordan curve. Within this region there are part regions. One can begin by distinguishing as such parts the "inner-personal" regions (*I*) from the motor and perceptual region (*M*). The motor and perceptual region has the position of a boundary zone between the inner-personal regions and the environment (*E*; Fig. 41). Two groups of facts stand in favor of such a representation.

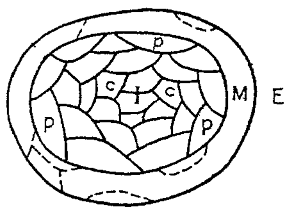


FIG. 41.—Topology of the person. *M* motor-perceptual region; *I*, inner-personal region; *p*, peripheral parts of *I*; *c*, central parts of *I*; *E*, environment.

1. Needs or other states of the inner-personal regions can influence the environment only by way of a bodily expression or a bodily action, that is, by way of a region which one can call the motor region. Koffka (47, p. 342) uses the term "the executive" for this region. According to the previous discussions of "paths of influence" (see p. 172) we have therefore to represent the motor region as a boundary zone between the inner-personal regions and the environment. One of the most important processes in this motor region is speech. It plays a great role in the communication between the person and his social environment. The use of gestures, "smiling at" and "looking at" belong here. The position of the motor region as an intermediate between the environment and the inner-personal regions holds for purposeful actions as well as for undirected affective discharges of tension, *i.e.*, for all changes of the environment *E* resulting from the state of the inner-personal region.

2. We find an intermediate region again when we consider the influence in the opposite direction, namely psychological

changes of the inner-personal region resulting from changes of the environment. This intermediate region corresponds to the perceptual system in the broadest sense of the word, that is, to hearing, seeing, etc. It is identical in part with the motor region. The eye for instance can both express and perceive. Other parts, like the ear, serve with man to transmit events only in one direction, from outside in. In any case the boundary zone between the inner regions and the environment includes both motor and perceptual systems.

It is to a certain degree arbitrary where one draws the boundary between the motor-perceptual system and the inner regions, whether for instance one considers the understanding of speech as an event within the boundary zone or within the inner-personal systems. The essential task is to determine the relative position of the regions in question, and the degree of communication between them and their neighboring regions. The same is true of the boundaries between the motor region and the environment. Both determinations depend upon the nature of the person and also upon the momentary state of the life space. During a medical examination the boundaries of the body are at the same time the boundaries between the person and the environment. But usually the clothing has to be counted as part of the person. The outer boundary of a child may be different when he is in contact with his mother and when he is with a stranger. In cases of embarrassment, for instance when one is suddenly exposed to critical glances of a stranger, the clothing and the whole appearance is often strongly emphasized and stands out as a special zone within the boundary region of the person. Under certain circumstances regions which are usually hidden can lie open or can be easily discerned through the surface layers.

With the motor-perceptual region, as with every boundary zone, one must consider its dynamic properties. There are, as we mentioned before, great individual differences which depend upon age and personality.

The difference between motor and inner regions is certainly not only a difference of position but is also a difference of function within the whole person. In a certain sense the motor systems have the position of "tools" of inner systems.

As an example of a disagreement between certain functional relations and the topology of the quasi-physical field we shall consider the relationship between mother and infant in the life space of the mother. It seems to me characteristic of this relationship that the mother picks the child up, lays him down, washes him, etc., without asking him. She uses direct bodily force in a dictatorial manner. Thus, the mother controls the infant by her will in a way which is only slightly different from the way in which she controls her own body.

On the other hand the actions of the mother are wholly at the service of the infant. She tries to act entirely according to its needs, *i.e.*, the needs of the child control the actions of the mother. Functionally therefore the needs of the child, as the mother understands them, have for her the position of an inner system, that is, of a system which directs the motor region *M* of the mother. It becomes clear that such a representation is meaningful when one considers that the birth of the child does not complete his psychological separation from the mother. The psychological separation in the sense of the freeing of the child from the mother is usually completed only much later. At the same time however the child has to some extent the position of an object in the environment or one may say it has the position of a part of her own body, but a part which is in direct contact with the rest only at certain times, for instance when the child is being fed. The antagonism between functional dependency and bodily separation leads to typical inner conflicts of the mother.

Within the motor-perceptual region one can distinguish again between more "peripheral" and more "central" regions. Such distinctions play an important role in the theory of perception (47; 46*a*; 60*a*). In so far as action is concerned the motor region seems to possess a relatively high unity: it is difficult to carry out four or five unrelated activities at the same time. It seems that the motor system can be connected dynamically with only one inner region or one relatively unified group of such regions at a given moment. If the motor system were to be guided by all the needs of a person at the same time, his behavior would become chaotic. The muscular tonus in one part of the motor region is closely connected with that in the others (2, 29*a*). The technique for psychodiagnostics, as used by Luria (63), seems to be based essentially on creating a close communication between certain inner-personal regions and a certain part of the motor region.

Central and Peripheral Inner-personal Strata.—Within the inner regions of the person one can distinguish between more central (*c*) and more peripheral strata (*p*, Fig. 41). The necessity for such a distinction showed itself in Karsten's experiments (40, pp. 236–237) on psychological satiation: actions which belong to more central strata are *ceteris paribus* more quickly satiated. It is of great general importance whether a psychological process belongs to more central or to more peripheral strata. Dembo's experimental investigations (20, pp. 101*ff.*) on anger have shown the significance of this factor for emotions. If only peripheral strata of the person are touched, manifestations of anger occur more easily. The outbreaks of anger are then more superficial. If more central strata are involved an open outbreak of affect is more rare. Indeed the boundary zone between the central strata (*c*, Fig. 41) and the environment (*E*) is stronger than the boundary zone between the peripheral strata (*p*) and the environment. Besides, the central regions may be surrounded by a specific functional wall (B_c, Fig. 42*a*). The peripheral strata come more easily into connection with the motor region to which they lie closer. Therefore expression usually occurs more readily when events of more peripheral strata are concerned. One speaks about personal matters only under special circumstances.

This is not only because the more central strata have on the whole less direct access to the motor region. Events in the opposite direction also, that is, from the environment to the inner regions of the person, usually reach the more central regions less easily. In conversation the way to the peripheral regions of the person is almost always open. But it is difficult to touch the real core of the person.

The relationship between the peripheral or the central position of an inner-personal region on the one hand and the degree of its accessibility and its ease of expression on the other hand, is not an entirely fixed one. It depends upon the momentary state of the person and upon the characteristics of the situation.

With some persons it seems to be easy to touch certain central places and to injure them like an "open wound." Some of these central regions seem to be always ready to communicate with the motor systems. Not less important than the topological position of the systems therefore are their dynamic properties and the dynamic properties of their boundaries. These are usually quite different for the different regions within one stratum and may change for the whole stratum. An example of a relatively simple change of dynamic relations

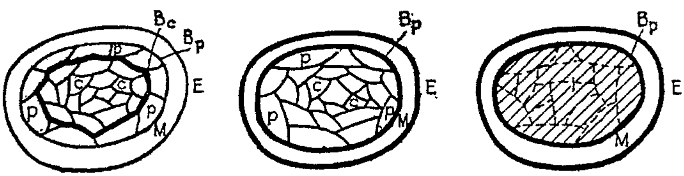


FIG. 42.—Relations between various strata of the person under different circumstances. (*a*) The person in an easy situation: the peripheral parts *p* of the inner-personal region *I*, are easily accessible from outside *E*; the more central parts *c* are less accessible; the inner-personal region *I* influences the motor region *M* relatively freely. (*b*) The person under stress, in state of self-control: the peripheral parts *p* of the inner-personal region *I* are less accessible than in (*a*); peripheral and central parts (*c* and *p*) are more closely connected; communication between *I* and *M* is less free. (*c*) The person under very high tension: unification (primitivation, "regression") of the inner-personal region *I*. *M*, motor-perceptual region; *I*, inner-personal region; *p*, peripheral parts of *I*; *c*, central parts of *I*; *E*, environment; B_c, dynamic wall between *c* and *p*; B_p, dynamic wall between *I* and *M*.

between the different strata is the transition from a state of superficial anger to a state of profound anger. When the person is in a quiet mood the boundary (B_p) between the peripheral strata and the motor region is dynamically relatively weak, but the boundary (B_c) between the peripheral and central regions of the person dynamically strong (Fig. 42*a*). If a situation of higher affective tension arises, the person usually replies with greater "self-control." To such self-control corresponds a greater separation of the peripheral strata from the motor region. At the same time the inner regions become relatively more unified (Fig. 42*b*). Dembo (20) has shown that if the affective tension is increased the resulting unifications can reduce the person to a more primitive level (Fig. 42*c*).

If the tension in the inner regions is still further increased it can break through to the motor region.

The separation of the inner regions from each other and their connection with the motor region can undergo changes of very different kinds. In the state of joy the inner-personal regions seem to be relatively unified and especially little separated from the motor zone. Joy expresses itself easily. Here again we find important individual differences.

Dynamically a more central position and greater tension of the inner systems are in many respects equivalent.

Individual Differences in the Structure of the Person.—At the present time we do not know very much about the kind of connection between the different systems and strata of the person. Yet the structure of the person shows considerable individual differences.

The Degree of Differentiation of the Person.—One of the most important dynamic differences between child and adult is that the person of the child is less differentiated into part regions. The growth of the psychological environment and of the person of the child does not mean simply quantitative increase in size, but it is at the same time essentially a process of differentiation (80; 48; 30, p. 8; 51, p. 206; 71, pp. 199–200; 36; 37) and, to some extent, of integration.

We cannot at this point discuss in detail the psychological problems of the processes of differentiation and integration but we have to ask what conceptual means are available for a comparison between the degrees of differentiation of different persons. One could say that the statement "the person *A* is more differentiated than *B*" is inadmissible for the following reason: it is a thesis about the number of part regions of a whole region, namely, the person, and it seems doubtful whether there is any value in comparing the number of part regions of two persons.

We distinguished between two possibilities of determining regions: the characterization by certain qualitative properties on the one hand and by locomotions or communications on the other (p. 94). If one uses the qualitative characterization, one can speak of different regions whenever one can make qualitative distinctions between regions. It is clear that this is an entirely relative standard, for what seems to be a homogeneous region at a superficial examination may show qualitative differentiations when one looks more closely. The number of distinguishable regions would thereby be made dependent on the degree of accuracy of the examination. This seems to make an objective comparison of the number of part regions impossible. For instance in both child and adult the first analysis shows the same number of regions, namely, a central and a peripheral inner-personal stratum and the motor stratum.

If one determines regions by means of the concept of dynamic communication, their unity is determined by their dynamic wholeness. But in this way also we achieve only a relative determination of the units of regions since we find different degrees of wholeness. It is for instance possible to consider child and adult each as one single dynamic region.

Another way of approaching this problem is suggested by our discussion of finitely and infinitely structured spaces. It might be possible to designate as "smallest regions" those regions within the person which at a given moment can no longer be broken up into psychologically meaningful part regions. As a matter of fact the assumption that such objectively not-further-structured dynamic unities are the structural elements of the person seems to be justified. Unfortunately at the present time a comparison between the degree of differentiation of different persons is not possible in this way.

Nevertheless, the dynamic connection can be used in determining the degree of differentiation of the person. Even if we can designate child and adult each as one single dynamic region, still the degree of wholeness of this system is greater with the child than with the adult: a change of one part of the system in the child usually influences all other parts to a much greater extent than in the adult.[1]

[1] E. Duffy (21, 22), for example, found that in adults the muscular tensions tend to be more differentiated among particular groups of muscles than in children.

We cannot discuss special problems at this point, especially not the difference

This degree of dynamic unity of the whole person can be taken under certain conditions as a criterion of the degree of differentiation into part regions, that is, if one takes as basic unities regions which show the same degree of dynamic separation from the neighboring regions.

The following consideration shows that we are justified in using the degree of unity of the whole region as a criterion of the degree of its differentiation into regions of a certain degree

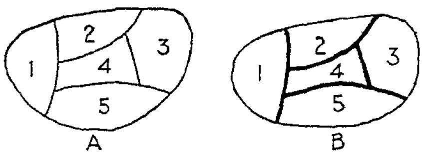


FIG. 43.—Two systems of the same structure and the same degree of differentiation but of different degrees of dynamic unity. The system *B* as a whole shows a less degree of unity than *A*. The parts 1, 2, 3, 4, 5, of the system *A* are less separated than the parts 1, 2, 3, 4, 5, of the system *B*. A given change of one of these parts (*e.g.*, 5) will therefore affect the other parts (1, 2, 3, 4) to a lesser degree in *B* than in *A*.

of separation. If within two whole regions neighboring part regions depend on each other to an equal degree, that whole region will in general be more strongly unified which contains fewer part regions. The reverse is also true: whole regions of the same degree of differentiation show a stronger dynamic unity if their part regions are less separated from each other. *A* and *B* may be two whole systems which are differentiated to an equal degree, *i.e.*, they contain the same number of part systems ($A = 1_A + 2_A + 3_A + 4_A + 5_A; B = 1_B + 2_B + 3_B + 4_B + 5_B$) (Fig. 43). To simplify the example we shall assume that the part regions are in themselves homogeneous and of the same quality in *A* and in *B*. The structure shall also be of the same nature in *A* and *B*. The only difference shall be that the part regions of *A* are separated from each other by less strong walls than the part regions of *B*. In this case *B* is dynamically a whole of a lower degree of unity than *A*, in spite of the fact that it has the same degree of differentiation.

in the kind of interdependency between certain systems in the child and in the adult, which may be as important as the mere difference in the degree of differentiation.

The statement that a more differentiated system shows a lower degree of unity, is not valid without exception, even if we presuppose the same degree of dependency of neighboring regions on each other. As a third factor we have to mention the kind of structure of the whole, that is, the special arrangement of the part regions. The degree of unity of the whole region can vary even when the number of part regions and the degree of separation of neighboring part regions are the same. This is shown for instance by a comparison of systems *A*, *B*, and *C* in Fig. 44. While there are in *A* and *B* part regions

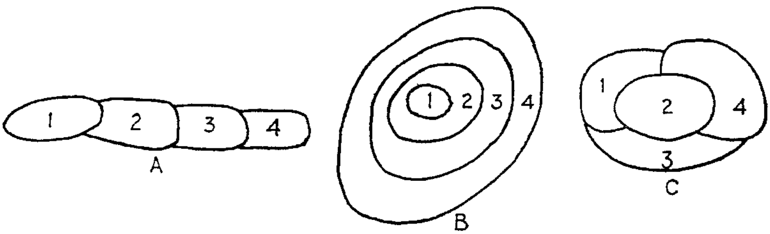


FIG. 44.—The dependency of the degree of unity of a system on its structure. The degree of differentiation (number of part systems) of the system *A*, *B*, and *C* and the degree of separation of neighboring part-systems are the same. Nevertheless the degree of unity of the whole system is different for *A*, *B* and *C*.

which can influence each other only through three separating walls, in system *C* we find never more than one wall between the part regions, in spite of the fact that the number of part regions is in all three cases the same.

It would be a difficult and important task of a "general gestalt theory"[1] to investigate how the degree of unity of a whole depends on its structure. The following proposition seems to contain a fundamental principle: the dynamic unity of a whole depends not only on the relation of the parts of the whole to each other but no less on the relation of the whole to its environment. As a rule a greater separation from the environment increases the inner unity of the whole.

In determining the degree of dynamic unity of the whole person one must therefore always take into account (1) the

[1] Köhler's investigations (43) are an approach to this problem. *Cf.* also Rashevsky (72).

degree of differentiation, (2) the degree of dynamic separation of neighboring part regions, (3) the special structure. If we assume that the structure is roughly the same in different persons, we have still to consider factors 1 and 2. It is however possible to separate these two factors if one succeeds in comparing the dynamic strength of the walls between the single part systems of the person concerned.

Such a comparison can sometimes be made. Investigations of satiation and substitution have shown that the psychological systems of certain feeble-minded persons are separated from each other by relatively strong and rigid walls. On the other hand it is characteristic of the sensitive problem child that there is only a small degree of separation of the part regions from each other. One can express this also in the following way: the person of the problem child corresponds to a more fluid, the person of the feeble-minded child to a more rigid material (52, pp. 209–210). The normal child is intermediate in this respect. The lesser dynamic separation of the systems in the problem child involves a closer connection between his central and peripheral strata. Therefore the condition of the central regions more readily expresses itself, for instance in a stormy affective outbreak. At the same time these expressions show a superficial character. The central strata of these children lie, from the point of view of dynamics, less deep; even if the degree of differentiation were the same as that of normal children, the boundary zone which lies between the central strata and the environment would sheath the central strata less than with the normal. Actually the degree of differentiation of the feeble-minded as well as of the problem child is less than that of a normal child of the same age. This shows itself in their infantilism and in the primitiveness of their behavior.

An observation of the process of differentiation in the development of the individual child shows that the older child is in general a more differentiated system. But the difference between adult and child, between children of different ages, and between the adult and the aged is certainly not only one of differentiation of the whole system. At the same time there is at least also a difference in the fluidity of the whole person; further a difference in the kind of structure (52, pp. 207*ff*.).

The degree of primitiveness of behavior seems to be a good symptom of the degree of differentiation of a person. Also the achievement of a person in an intelligence test seems to depend above all on the degree of differentiation of the person, or at least on the differentiation of certain part regions.

The influence of malaria treatment in cases of general paralysis as shown by Galant-Rattner (29), is an example of change in the state of the person.

The Kind of Structure and the Function of the Part Regions.—Individuals differ not only in the degree of differentiation of the whole person but also in the way in which the different part regions are arranged, in the strength of the connections between the different individual part regions, and the function which they have in the life of the person. We shall call these characteristics the "kind of structure of the person."

Within the same person the different part regions are not differentiated to the same degree. This is obvious for the different regions which are connected with knowledge and skills. Individuals also differ markedly as to which part regions are highly and which are poorly differentiated. We find similar differences in kind and degree of differentiation in non-intellectual regions, for instance in those inner-personal regions which are connected with family, friendships, or occupation. With the feeble-minded certain "irreal" regions (see p. 196) which are important for phantasy seem to be relatively little differentiated (51). Or again if one speaks of a "harmonious character" it is meant that the different part regions of the person are relatively well balanced.

The functional significance of such regions can be very different. The region related to occupation for instance can play either a fundamental or a more secondary role in the structure of the person. It can have its source in very different needs. The significance which a certain activity has for a person and the satisfaction which it offers him depend on the functional significance of this region in the life of the person.

The degree of dynamical connectedness of the different parts of the person can be nearly equal within the whole region of the person, or certain regions can separate themselves to an especially high degree from the others and develop relatively independently. This can be observed in the normal person and it seems to be important for certain mental diseases.

In those cases in which Freud speaks of "complex" and McDougall of "dissociation" (65*a*) there is probably also a considerable degree of isolation.

A person's structure is often relatively constant over a long period of time. Nevertheless a great change in the environment, falling in love, being "converted," or some other decisive event can bring about a far-reaching change of structure which may be either temporary or lasting.

The question of the structure of the person has special bearing on developmental psychology. For the structure of an individual at any given time is a product of his development. The differentiation of the whole person, connectedness, relative differentiation, and function of the single part regions seem to undergo typical changes during individual development (26). The investigation of these processes about which we know very little will be possible only along with the investigation of the general laws and only if one carefully considers the different conceptual problems which we have discussed.

The Connection between Dynamic and Topological Factors.—In concluding our discussions of the topology of the person we shall treat somewhat more extensively an example which throws light upon the connection between the dynamic and the topological properties of personal regions. This dependency becomes especially clear if one considers the momentary changes of personal structure.

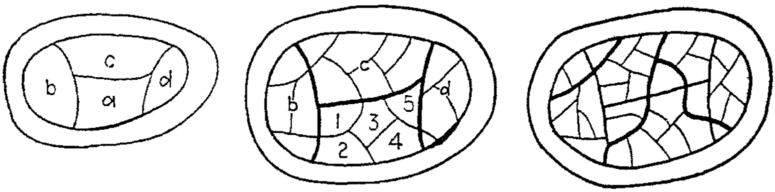
We have already mentioned that according to the results obtained by Dembo a state of great inner tension brings about a dynamic unification of the person. This greater unity rests at least in part on the fact that the degree of separateness of the part systems is a relative value and has to be related to the magnitude of the relevant forces. Dynamic walls which may suffice to separate part systems to a high degree, if the forces in question are small, can become relatively unimportant in the face of the strong forces which arise during a state of high tension. For this reason a great increase in inner tension means *ipso facto* a dedifferentiation of the person.

Certain findings of Coghill (17) in regard to the development of behavior of embryos may be explained in a similar way. These experiments show that behavior must not be characterized as a gradual genesis of more inclusive reactions out of what are at first isolated reflexes. Rather the first responses to stimuli are undifferentiated reactions of the whole organism. Only gradually more differentiated responses of single parts of the whole organism come into being. The development of embryonic behavior like the psychological development from childhood to maturity (see p. 203) can show besides differentiation some integration (13; 14; 71, p. 200).

The differentiation occurring in development does not entirely destroy the original unity of the fundamental dynamic systems. This is shown by the fact that the embryo, having attained a certain degree of differentiation, drops back to a more primitive level when the connection with the blood stream of the mother is severed or the fetus becomes fatigued;[1] that is, it again responds to a stimulus with a more undifferentiated whole reaction. We find here therefore a temporary "regression" like that which occurs when a child is sick or in the tension of anger. In explaining this fact one usually points to the proposition that differentiation of reactions results from an inhibition (69, p. 43; 3, pp. 418, 442) of the original process by newly developed centers. In dynamic terms this means that the differentiated reaction is caused by an opposition of forces. It may however be more simple to bring these processes directly into connection with the fundamental fact that the embryo gradually differentiates itself into regions which dynamically are relatively separated.

[1] ANGULO Y GONZÁLES, A. W. (3), p. 420. The experiment was made with embryos removed, at different ages, from the uterus in such a way that the connection with the blood stream of the mother was kept intact.

That the embryo responds with undifferentiated whole reactions implies that certain inner systems or the motor strata[1] are unified to such a high degree that a stimulus produces a reaction of the whole body. *a* (Fig. 45*a*) may be the system whose lack of differentiation is responsible for the reaction of the whole body to the stimulus concerned. (*a* may be identical with the whole organism or with one part of it.) The later differentiation of the reaction may rest on the fact that *a* has differentiated itself into the part regions 1, 2, 3, 4, 5 (Fig. 45*b*).



(*a*) (*b*) (*c*)

FIG. 45.—Regression and the structure of the person. (*a*) Relatively undifferentiated state of development (the whole systems contain relatively few part-systems); the regions *a*, *b*, *c*, *d* are separated. (*b*) Later state of development of the same individual: regions *a*, *b*, *c*, *d*, show further differentiation into sub-regions; the newly developed part regions 1, 2, 3, 4, 5 of region *a* are separated by weaker walls than the older regions *a*, *b*, *c*, *d*; under the stress of relatively strong forces these stronger walls (corresponding to the older primitive structure) become again the main determinants and result in "regressive" ("infantile") behavior (similar to that in the stage (*a*)); this holds also when the person becomes very fluid. (*c*) Differentiation with change of the basic structure: certain newly developed walls between part regions have become stronger than the older partitions; in this case a primitivation would lead to a behavior somewhat different from that which occurs in a case of primitivation of (*b*).

The separation of biologically "young" parts from each other is usually less rigid than that of older parts. The dynamic walls between the regions 1, 2, 3, 4, 5 are therefore to be represented for this time as less rigid than the walls between *a* as a whole and its neighboring regions *b*, *c*, *d*. From this assumption, which biologically is at least very probable, there follows:

1. If the intensity of the stimulus is increased, the rigidity of the new walls will not suffice to keep the part regions as relatively independent systems in the face of the stronger influence. (This follows from the general relation between the degree of separation and the magnitude of the effective forces.) Therefore again there will be a reaction of the whole body. As a

[1] Either of these assumptions would suffice.

matter of fact one can attain a pronounced reaction of the whole organism even with adults if the stimulus which is chosen is strong enough. It is characteristic of responses to very strong stimulation, for instance to shock, as well as for the responses of undifferentiated organisms, that they are relatively independent of the point of application of the stimulus (3, p. 434). This results directly from the fact that the strength of the walls is only relatively determined.

2. A similar effect must occur if one, instead of increasing the magnitude of the forces, brings about a greater fluidity of the systems concerned. Each change of state of the organism in the direction of greater fluidity or lability will probably affect the young walls more strongly than the older, already more rigid ones. Psychological experiments with adults indicate that such an increase in the fluidity of the whole system occurs during fatigue (92, pp. 65–70). What changes of state bring about a dedifferentiation of the systems depends on the dynamic properties of the factors which separate the part systems and on the kind of influences in the face of which the separation should be effective. It is however at least not improbable that, in a fetus too, fatigue leads to an increase in the lability of the whole organism, and therefore weakens the relatively new differentations to a higher degree than the older ones.

3. The hypothesis just presented allows us to determine under what conditions a dedifferentiation will not lead to behavior which corresponds to an earlier state of development. We have assumed that this correspondence of behavior rests on the fact that the later developed part systems are separated from each other by more plastic walls than the older ones. But it can happen that the younger walls become in the course of time more rigid than those which were developed earlier. This changes the structure of the larger systems and the relationship of the single part systems to them. If at this moment a situation occurs which produces a temporary dedifferentiation of the person, the behavior will become again more primitive, but it will not be the same as the behavior at the earlier stages of development. For the remaining units of system will differ

essentially from the original ones (Fig. 45*c*). As a matter of fact we find, in spite of all similarities, considerable differences between the behavior of an infantile adult and that of a child. This difference is probably not caused merely by a difference in the properties of the material of the system but also by a difference in their structure.

The dynamic structure of the organism can be altered not only by changes in the relative strength of the walls by which the different part systems are separated from each other; under certain circumstances existing walls may actually be weakened or destroyed.

Integration and Dedifferentiation

Besides the process of differentiation there seems to occur a process of "integration" during development (McDougall, 65*a*).

This process counteracts the process of differentiation insofar as it creates a greater degree of interdependence of the different systems of the person and in this way makes for a higher degree of unity of the person as a whole. Mathematically this integration could be viewed as a reversal of the differentiation so that there would be no difference between the dedifferentiation occurring under the specific (more or less "abnormal") conditions discussed above the normal process of developmental integration.

Dynamically speaking, however, these processes seem to be definitely different in character. The integration of the person during development seems not to be a strict reversal of the differentiation and also not a simple restructuring of the inner personal system. It is rather a process by which a certain system (or group of systems) becomes "dominant" insofar as it is able to impose certain patterns of action and to build up certain quasi needs. I am inclined to venture the theory that we have to deal here with the relation of an inducing field to an induced field (Lewin, 52, p. 97). In other words, the position of the dominant system is dynamically equivalent to that of the center of a social power field. The same type of dynamic interdependence is probably characteristic for the relation between the inner psychological region and the motor region or between a person and a tool. It differs from that interdependence of neighboring tension systems which we discussed previously.

These problems can be considered in detail only in vector psychology. However, it might be well to limit the term "integration" in psychology to those cases where the unification of differentiated regions is due to the establishing of the hierarchic relation between an inducing field and an induced field. A unification due to destruction (or weakening) of the dynamic walls that separate systems might be called "dedifferentiation" as opposed to "integration."

D. DIMENSIONS OF THE LIFE SPACE

CHAPTER XVIII

THE DIMENSIONS OF THE LIFE SPACE

In our figures we have used so far a plane, that is a two-dimensional space, for the representation of psychological fields. One can raise the question: is it correct to use such a manifold for this representation? In other words, how many dimensions has the life space?

Mathematics of Dimensions

Mathematics has only within recent years found a way to treat problems of dimension satisfactorily. Differences in dimension are not differences in size or in potency of the space.[1] One can coordinate one to one the set of points of a line to the points of a limited two-dimensional region or of a three-dimensional body. In considering how many dimensions one ought to attribute to the life space, one therefore does not have to take into account the purely quantitative question of the space "available" in the representation.

Mathematics shows that dimension is a property of the "inner structure" of the space, a property which is closely connected with topological characteristics. It is characteristic of a two-dimensional space, for instance a plane, that within it there is no possibility of connecting each of five or more points with each other such that the connecting lines do not intersect. Further, as we have said, in a two-dimensional space it is impossible to connect a point within a circular area with a point outside of it, without intersecting the boundary of the area.

[1] For the following discussions *cf.* K. Menger, (66), pp. 1*ff.* In this book one finds also remarks about the historical background, pp. 83*ff.*

In a three-dimensional space such a path is possible. These facts show clearly how important the number of dimensions of psychological life space is, for instance, for the question of what locomotions are possible.

The determination of the number of dimensions in modern theory of dimension rests essentially on a general relation between a spacial object and its boundary. The boundaries of a three-dimensional body for instance are surfaces, that is, they are themselves two-dimensional. A surface is limited by lines, that is, a two-dimensional space is limited by one which is one-dimensional. The one-dimensional finite line is limited by points which themselves have the dimension zero. In general one can say: the boundary of an n-dimensional space is $(n - 1)$-dimensional (66, p. 80; 79, pp. 207–208). It will be clear, without going into the mathematics of these problems in detail, that on the basis of this fact a procedure of recursion is possible. One can begin with an empty set as a (-1)-dimensional space and from it proceed to the higher dimensions.

From the point of view of mathematics there is no reason to limit the number of dimensions to three. The progression to spaces of n dimensions is possible without difficulty. Mathematics deals also with spaces whose number of dimensions is different at different points. It might seem enticing to psychology to make free use of the possibilities which the introduction of a more complicated space or of a space of many dimensions would offer. In the beginning however it is important for reasons of discipline and economy in making theories to introduce no more dimensions than are absolutely necessary. Therefore our problem has to be formulated as follows: what is the minimum number of dimensions that is required to represent the life space?

It may seem advisable to distinguish our problem from another in which it is also possible to speak in a mathematical sense of dimensions. One can think of the different properties of an object or a system as variables and can represent them by means of a system of coordinates which has as many dimensions as there are distinguishable properties. Physics speaks in this sense of a phase space. (In the same way time can be represented as a fourth dimension.) Similarly Boring speaks of dimensions of the stimulus (6) in psychology. It should be clear that the question of the number of dimensions of the life space is entirely different from the question of the number of distinguishable "properties" of psychological objects or events.

The Dimensions of the Psychological Environment

Reality.—In the beginning of our experimental work two-dimensional spaces sufficed for the representation of situation. For some time however we have been using three-dimensional spaces.

In determining the number of dimensions of the life space it is useful to refer to the concept of locomotion and the above mentioned relations between the dimension of a region and its boundary. That the number of dimensions of a life space is greater than zero results from the fact that movement is impossible in spaces of zero dimension. The fact of locomotion means that the life space is at least one-dimensional.

The boundaries of a one-dimensional space are themselves of zero dimension; they correspond to points. Is the point an adequate representation of the boundaries of psychological regions? To represent the life space as one-dimensional, that is, as a group of lines, seems at first thought to contradict the facts. Nevertheless, it is not easy to reject such a representation in a mathematically incontestable way. For one-dimensional spaces can consist of very complicated networks. For instance, one can define one-dimensional spaces in such a way that it is possible to distinguish at each point any number of directions. Nevertheless, I believe that the nature of the psychological processes which actually occur makes it impossible to represent the life space as one-dimensional. It is often possible to go "along a boundary" in the psychological field. Furthermore, one can often distinguish different sectors as parts of a connected boundary. It is, as we have mentioned, important for the behavior of a person that the different parts of a boundary can differ in their strength. These facts show that there are boundaries which do not consist of one point, but which consist at least of lines, that is, of one-dimensional manifolds. And a boundary which has at least one dimension implies that we are dealing with a space which is at least two-dimensional.

It is more difficult to answer the question whether one should use still more dimensions. When we are dealing with quasi-physical fields, it may be necessary to represent the psychological field as three-dimensional space corresponding to the three-dimensionality of physical space. To circumvent, *e.g.*, a two-dimensional barrier, certain locomotions are possible between the points of a three-dimensional physical region that would not be possible in a two-dimensional space. From the properties of these locomotions one may be able to demonstrate that the quasi-physical field, like the physical space, has also three dimensions.

Degrees of Irreality.—A two-dimensional representation of the quasi-social and quasi-physical fields has sufficed for most of the psychological problems we have treated so far. It is however necessary to make the transition to a further dimension when we have to distinguish between different "degrees of reality." A daydream, a vague hope, has in general less reality than an action; an action sometimes has more reality than speech; a perception more than an image; a faraway "ideal goal" is less real than a "real goal" that determines one's immediate action (34; 52, p. 250). Action itself can be of very different degrees of reality. Processes which concern strong needs of the person and in which he has to surmount strong physical or social barriers have usually a high degree of reality. Among the quasi-conceptual processes one can for instance distinguish carefully planned consideration of the ways and means which lead to a certain goal from free play of phantasy, which is more irreal (64; 20).

It is important to distinguish the concept of degree of reality in psychology from the epistemological concept of reality. For the epistemological concept there are no different degrees of reality. An intermediate between existence and non-existence is not possible. The psychologically "irreal" objects are real for psychology in an epistemological sense; that is they exist, they have effects. It must further be emphasized that the physical reality of the object concerned is not decisive for the degree of psychological reality. For the primitive or the child living in a "magic world" those facts which are believed are real (64).

Experiments have shown that the degree of reality is a very important dynamic property of almost all psychological objects and processes. This has come out especially in experiments about the level of aspiration (34), the origin and effects of substitute action (64, 81), the formation and change of goals (34), emotional processes (20), memory (11), and play (81).

We have found it necessary to use a special dimension for the representation of degrees of reality. If one represents the totality of real regions as two-dimensional space, for instance as a plane, one has to coordinate to the different degrees of reality different planes which lie one above the other. In demonstrating mathematically the necessity of such a representation, one will have to consider that locomotions are possible within the same degree of reality and between regions of different degrees of reality. The totality of what belongs to the same degree of reality, for instance the world of wishes of a person at a certain moment, is itself a region. In it one can distinguish part regions. We can speak of movements in the irreal strata as in the stratum of reality. Therefore it is possible to determine the topology of these regions. The topological structure of a stratum of irreality can be similar to that of the stratum of reality. Under certain circumstances however its structure is typically different from the structure of the plane of reality, especially if the situation in the plane of reality is very disagreeable for the person (52, pp. 146*ff*.).

In following the principle of using no more dimensions than necessary to represent the life space, one will have to ask whether or not it is possible to represent the irreal levels without introducing a new dimension. A closer examination shows that such a representation is not permissible.

Two methods which are somewhat alike suggest themselves if one tries to represent regions of a lesser degree of reality without using a further dimension. One could attempt to indicate levels of successively decreasing reality as a chain of regions in such a way that to each region corresponds a lesser degree of reality than to the one preceding it (Fig. 46*a*). Or one might represent them as a series of ring-like regions each of which is entirely enclosed by the succeeding one (Fig. 46*b*).

However, such a method would not permit an adequate representation of certain locomotions. This becomes clear if one distinguishes between the single part regions within the same level of reality. a^1, b^1, c^1, d^1, e^1, . . . may be part regions of the degree of reality R^1; a^2, b^2, c^2, d^2, e^2, . . .

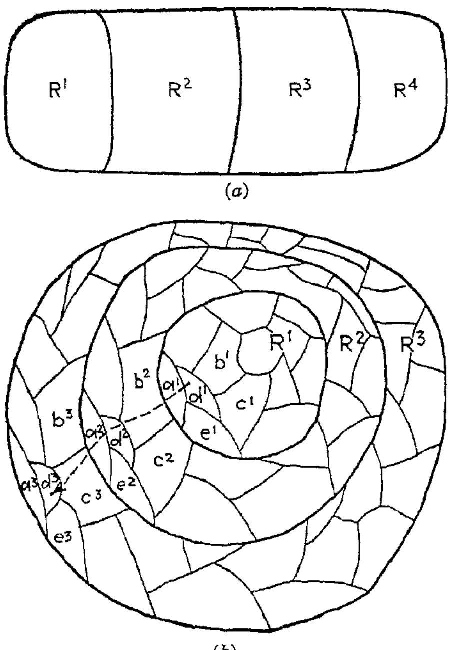


FIG. 46.—Attempt to represent regions of different degrees of reality without introducing an additional dimension. The different degrees of reality are represented in (*a*) as a chain of regions, in (*b*) as a series of ring-like regions R^1, R^2, R^3, R^4 ($R^1 > a^1, R^1 > b^1, R^1 > c^1, R^1 > d^1; R^2 > a^2, R^2 > b^2, R^2 > c^2, R^2 > d^2; R^3 > a^3, R^3 > b^3, R^3 > c^3, R^3 > d^3$); the regions d^1, d^2, d^3 correspond with respect to their contents.

may be part regions of the degree of reality R^2; a^3, b^3, c^3, . . . may be part regions of the degree of reality R^3. The difference in the degree of reality may not be very great and the structure of the different levels of reality may be roughly similar. Furthermore, regions d^1 in R^1, d^2 in R^2, and d^3 in R^3, may belong together psychologically as far as their content is concerned.

The two-dimensional representation has the disadvantage that a movement within the same level of reality, for instance from b^1 to a^1, almost always implies an approach to or a withdrawal from regions of a higher or lower degree of reality. Furthermore, there is the difficulty that if one represents a locomotion from one region into another region of corresponding content but lower degree of reality, it must cross other regions of the same degree of reality first. For instance one has to cross the regions a^1, b^2, d^2, a^2, c^3, in order to go from d^1 to d^2 and d^3. Within the actual psychological life space too one cannot move directly from every region of one degree of reality to every region of another degree of reality. But certainly there occur psychological locomotions which pass a whole series of different degrees of reality within a region of the same content. If one represents the levels of different degrees of reality as a two-dimensional space these locomotions cannot be represented as continuous paths without falsifying the relations of connectedness of regions within the different levels. This shows that one has to introduce a new dimension for the representation of differences in degree of reality if one wants to represent such locomotions adequately.

Incidentally these considerations also make clear a fundamental mathematical characteristic of dimension. Points or other part regions of spaces of different numbers of dimension can be coordinated in one-to-one correspondence only when one destroys the topological relations: the two-dimensional representation (Fig. 46*b*) destroys the actual existing connections between d^1, d^2, d^3.

Since spaces of no less than two dimensions correspond to regions of the same degree of reality the life space has to be represented by at least three dimensions (Fig. 47) if one takes into account the different degrees of reality. We coordinate to the different degrees of reality planes or levels which lie one above the other. In our diagrams we shall represent levels of higher degree of irreality (*I*) by planes which lie higher. One could of course also do the reverse.

One of the most important dynamic differences between different degrees of reality is the greater fluidity of the more irreal levels (*I*). This greater fluidity shows itself in different facts: (1) Barriers in the environment offer relatively little resistance (one can do what one wants to do in irreality (20, pp. 36*ff*.)); (2) the boundaries of environmental regions can be shifted more easily and are less definitely determined (81, p. 149); this holds also for quasi-conceptual regions of different degrees of reality; (3) a diffuse discharge of an inner-personal tension system occurs more quickly (11, p. 2); this corresponds to weaker walls of the inner systems; (4) the boundaries between person and environment are less clear and the structure of the environment depends to a greater extent on the needs of the person (52, p. 146).

Problems of Dimensions of the Person

The question arises how many dimensions has the region which represents the person within the life space. It would be

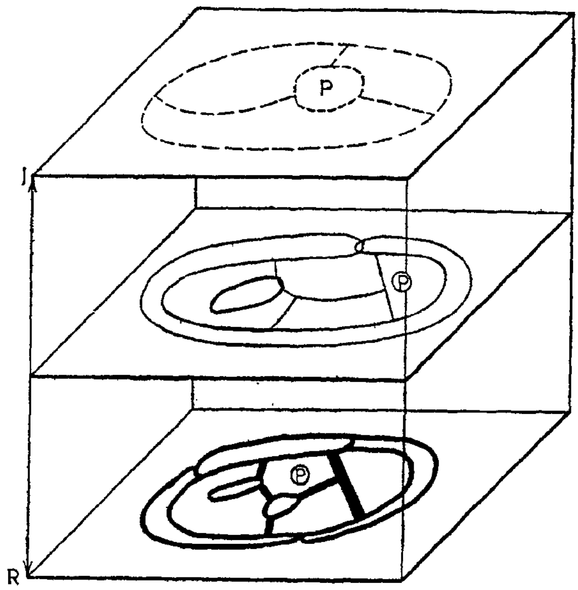


FIG. 47.—Representation of the different degrees of reality by an additional dimension of the life space. *R*, more real level; *I*, more irreal level; *P*, person. In a level of greater reality the barriers are stronger and the person *P* is more clearly separated from his environment.

wrong to represent the person only in the plane of reality. For he can act also within the more irreal levels. Furthermore, there seem to exist inner-personal systems which one has to coordinate to irreal levels.

This problem raises questions to which it is hardly possible to give definite and detailed answers at the present time. One certainly has to represent the transition from sober reasoning to phantastic attempts to solve a problem as a transition to processes within irreal levels. But are we dealing in these cases with locomotions of the person as a whole? If the person were at each moment within only one level of reality, one would have to represent him as two-dimensional. However, such a thesis is open to doubt since the person, even during the flight into irreality, stays within the level of reality in part at least, namely, as a bodily being. This is proved by the fact that he can be influenced by processes within this level even if he is somewhere else in his phantasy. There may be cases in which the possibility of influence is so weakened that one has to represent the person as two dynamically relatively separated regions which belong to different levels of reality. Such a condition may sometimes be realized in schizophrenia.

One might also think of representing the person as a region which belongs at the same time to more than one level of reality. In this case the person like the environment would have to be represented as a region of at least three dimensions. It may sometimes reach farther, sometimes less far into the different planes of reality and the "center of gravity" of the person may belong to different levels at different moments.

Finally the transition to levels of lesser degree of reality could in most cases be thought of as a locomotion of parts of the person. Such a locomotion of a part region of the person must not necessarily lead to a breaking up of the unity of the person. Assuredly we find such movements within the level of reality.

On the whole it seems to me that the psychological facts speak for attributing to the person the same number of dimensions as to the life space as a whole and for representing the person in all the levels of reality which the life space concerned possesses. It often is possible to represent the transition of the person to other levels of reality as a change in the relative weight (see p. 137) of the different levels of reality. It is, for instance, possible to treat the different levels of reality, R_1, R_2, R_3 . . . , in which the person P is at the same time, as an overlapping situation (see p. 138, $P < R_1, P < R_2, P < R_3, \ldots$). The behavior of a person as a whole is influenced at a given time by the various levels of reality to a different degree. (The relative weight W_{R1} of R_1 may be 70 per cent, of W_{R2}, 5 per cent, of W_{R3}, 10 per cent, etc.) These relative weights of the various levels differ under different circumstances and for different persons. A "flight into the irreality" would mean that the relative weight of certain levels of higher reality (*e.g.*, W_{R1}) is diminished and that the weight of certain more irreal levels of the life space (*e.g.*, W_{R4}) that may have been zero or only slightly greater than zero has increased considerably (*e.g.*, $W_{R4} = 40$ per cent instead of $W_{R4} = 5$ per cent as at first). Such representation of a "flight into irreality" avoids many difficulties which a representation by locomotion of the person from one level of reality to another naturally shows. It has also the advantage of emphasizing a change in the importance of the various regions within the person, which is characteristic of such transitions. Certainly such change is not only structural but functional as well.

It is relatively easy to distinguish between person and environment within a level of high reality. (We shall at this place disregard the question of the accurateness of the determination of boundaries.) In any case it is not difficult to show that there are places in the environment which the person does not occupy. In levels of great irreality however it is often uncertain whether one should designate a certain region as part of the person or as part of the environment.

The stratification of person and environment into levels of different degrees of reality must not be confused with the distinction between central and peripheral strata within the person. For within the same level of reality one will have to discriminate between more central and more peripheral inner regions. On the whole, processes within the more irreal planes seem to have a closer dynamic relation to the core of the person and to his central needs. It is a special problem whether one has to represent the motor stratum only within the level of reality.

The Differentiation of the Life Space in the Dimension Reality-irreality

It is not only when phantasy is especially active that the life space possesses a stratification into regions of different degrees of reality. This is always the case. The degree of this stratification however depends upon the situation as a whole (81, p. 150). In some cases the transition of the person to levels of a different degree of reality is an extension or contraction of the life space (that is, of person and environment) in the reality-irreality dimension rather than a locomotion within an otherwise constant field. Moreover, the relative weight of levels of different degrees of reality can change.

It seems to be very different with different persons and at different moments how far the life space is extended in the direction toward irreality. One may ask whether there exist intrinsic limits of the life space in this direction, that is, whether there exists a plane of greatest irreality. It would hardly be possible at the present time to justify such an assumption.

There is much that speaks for the assumption of a level of greatest reality. One could attempt to establish the thesis of the existence of such a plane of "full" reality in each life space by calling attention to a special relation of the motor stratum to reality (see p. 177). In addition we might be tempted to connect conceptually the full reality with the objective physical or social influences from the "outside" on the life space (see p. 72). Reality is essentially characterized, as far as dynamics is concerned, by its independence of the will of the person. Nevertheless, there are facts which run counter to the supposition of an absolute level of full reality. Someone may believe that he lives in full reality until events of a still harder reality teach him better.

The facts which speak for a relativity of the plane of highest reality become still more significant if one thinks of the development of the life space from childhood to maturity. We have already mentioned that this development is to be characterized by an extension and especially by a differentiation of person and environment. Such a gradual differentiation of the life space can also be observed in the dimension of differences of reality. A great number of psychological facts lend support to the assumptions: (1) that there are only slight differences in degree of reality within the life space of the small child (Fig. 48*a*); (2)

that the levels within the life space of the small child correspond on the whole to an intermediate degree of reality (80; 70; 42, p. 706; 52, p. 104; 48, pp. 366*ff.*; 39) (*cf.* Fig. 48*a* and *b*). That means that the levels of reality of the small child are to be characterized as relatively irreal in comparison with those of the adult, and that the levels of irreality of the child are comparatively real (81, p. 162). It seems therefore advisable not to introduce the concept of a plane of "absolute reality." We

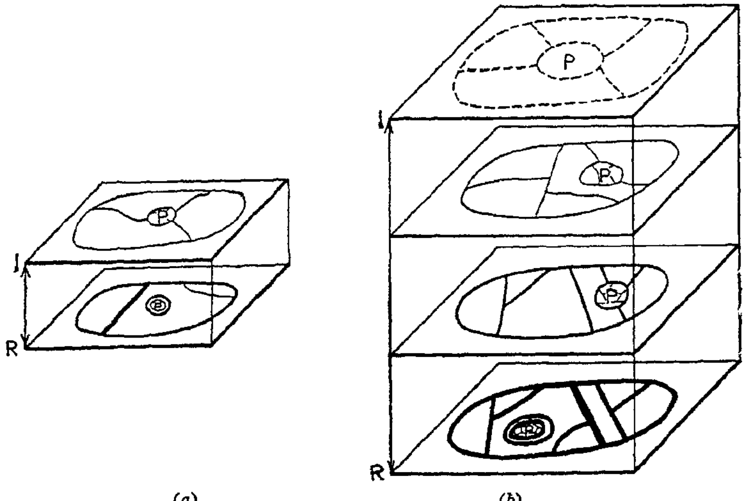


FIG. 48.—Life space of a child (*a*) and of an adult (*b*). The life space of the adult is more highly differentiated in the dimension reality-irreality. The range of reality and irreality layers in the life space of the child corresponds to an intermediate range of layers in the adult's life space.

have already mentioned that for different individuals and for the same person at different moments of time the degree of differentiation of the life space into levels of different degrees of reality varies. Further investigations are necessary before one can answer these problems definitely. But the concepts and methods discussed will, I think, suffice to meet any conceptual difficulties in representing the levels of irreality within the topology of the life space.

The concepts which we have developed in this book concern the whole psychological life space, that is, person and environment. They allow treatment of all problems of position and connections of the life space and its parts. They are applicable to quasi-physical as well as to quasi-social and quasi-conceptual facts. By means of these concepts one can represent the structural changes of person and environment and all kinds of locomotion. To a certain extent one can also deal with those problems of psychology that are dynamic in a narrower sense of the word, for instance the friction of a region, the solidity of a barrier with respect to locomotions, the degree of separation of regions with respect to communications of different kinds, and the degree of wholeness (dynamic gestalt) of systems of the environment and the person. Finally one can treat certain problems of tension and changes of state of regions, for instance the liquidity or solidity of a region.

But these dynamic questions are connected in one way or another with problems which go beyond topological psychology. Topological psychology can represent only the framework of events which are "possible" in a life space. In order to determine which events actually occur and what conflicts underlie them one has to take into account directed magnitudes, especially the concept of psychological force and the concept of field of force. The relevant concepts and coordinating definitions are to be treated as part of vector psychology.

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GLOSSARY

The concepts contained in this glossary are grouped as follows: (I) methodological concepts, *i.e.*, concepts related to epistemology, logic, theory of science, and methodology; (II) concepts of topological psychology. This group contains (*A*) mathematical concepts; (*B*) psychological concepts. In this latter subgroup are placed (1) concepts by which directly observable facts are coordinated to topological concepts; (2) dynamic concepts. This classification is not intended to be strict, but may aid in clarifying the logical position of the various concepts.

I. METHODOLOGICAL CONCEPTS

Approximation, method of: This method determines first the structure of the life space as a whole and proceeds gradually by determining more and more specific properties to greater exactness.

Cause: One has to distinguish historical and systematical causation. (*a*) Systematical causation: An event is considered as a function of the total situation at a particular time. The cause of an event is always the interrelation between several facts. (*b*) Historical causation: An answer is given to the question why an individual situation at a certain historical time and at a given geographical position has these particular properties.

Concepts: One can distinguish (*a*) mathematical concepts like boundary, region, vector; (*b*) dynamic (conditional-genetic) concepts like force, tension, resistance, fluidity.

Concrete: That which has the position of an individual fact which exists at a certain moment. In psychology the concrete can be represented as a part of the life space or as a property of such a part.

Construct: A dynamic fact which is determined indirectly as an "intervening concept" by way of "operational definition." A construct expresses a dynamic interrelation and permits, in connection with laws, the making of statements about what is possible and what is not possible.

Definition, coordinating: Certain observable facts are correlated to certain mathematical concepts. A coordinating definition should be univocal and if possible reversible.

Dimension of properties: One can think of the different properties of an object as variables and can represent them by means of a system of coordinates which have as many dimensions as there are distinguishable properties. Such a system of coordinates is sometimes called phase space.

Dynamic: Facts or concepts which refer to conditions of change, especially to forces, are called dynamic. Dynamic facts can be determined indirectly only (*see* Construct).

Existence: That which has effects. The existence and the time index of a psychological fact are independent of the existence and the time indices of the facts to which its content refers.

Explanation: Representation of a concrete situation in such a way that the actual events can be derived from it with the help of general laws.

Historic process (dialectic): Actual order of occurrences in the given world. Generally periods of apparently continuous transformation are followed by periods of crisis involving sudden changes of structure.

Influences "from outside": Influences on a life space which cannot be derived by psychobiological laws from the psychobiological properties of the preceding situation.

Law, empirical: A law defines the functional relationship between various facts. These facts are conceived as types, *i.e.*, historical time indices do not enter a law. A psychological law can be expressed by an equation, *e.g.*, of the form $B = f(L)$. The laws serve as principles according to which the actual events may be derived from the dynamic factors of the situation.

Phenomenal fact: Fact which can be observed directly.

Prediction: Only if, in addition to the law, the special nature of the particular situation is known is prediction possible.

Reality: The epistemological concept of reality (*see* Existence) is to be distinguished from the psychological concept of "degree of reality," which refers to certain strata of the life space.

Representation: Conceptual image (in the mathematical sense) of facts. Scientific representation presupposes (1) concepts which permit logically strict derivations and (2) coordinating definitions between concepts and observable data.

Space, mathematical: *see* Mathematical Concepts.

Space, phase: *see* Dimension of properties.

Space, physical: Space which includes the totality of all physical facts that exist at a certain time (the whole physical world).

Space, psychological: *see* Life space, in Psychological Concepts.

World, physical: Totality of more or less interdependent physical facts. All physical changes are the result of conditions or changes within one connected physical space. According to physics there are no influences upon physical objects "from outside" this space.

World, psychological: Totality of more or less interdependent psychological facts. There exist a plurality of psychological worlds, corresponding to a plurality of not connected psychological spaces (life spaces). These worlds are influenced "from outside."

II. CONCEPTS OF TOPOLOGICAL PSYCHOLOGY

A. Mathematical Concepts

Boundary point: Any surrounding of a boundary point of a region contains points which do not belong to that region.

Cut: A path which connects two boundary points of a region and which, aside from these boundary points, lies wholly within the region.

Dimension: The boundary of an n-dimensional space is $(n - 1)$-dimensional. Points of spaces of different numbers of dimensions can be coordinated to each other in one-to-one correspondence only when their topological relations are destroyed.

Intersection: The intersection or common part of regions A and B ($A \cdot B$) is the totality of points which are part of A and of B.

Jordan curve: A topological image of a circle.

Path: Connection between two points by a Jordan arc.

Region, closed: A region which includes its boundary points.

Region, connected: A region every point of which can be connected with every other point by a path which lies entirely within the region.

Region, foreign: A is foreign to B if the intersection of A and B is empty ($A \cdot B = 0$).

Region, open: A region for every point of which there is a surrounding that lies entirely within the region.

Region, simply connected: A region whose connectedness is destroyed by one cut.

Space, mathematical: Does not refer to any particular entities but only to relations. For these relations certain axioms hold.

Space, metrical: To any two points of such space a distance is coordinated for which certain axioms hold, especially the triangle axiom.

Space (region), structured finitely: A space (region) which can be divided into distinguishable part regions, but which is not infinitely structured.

Space (region), structured infinitely: A space (region) whose part regions can be divided infinitely into further parts.

Space (region), unstructured: A space (region) which cannot be divided into distinguishable parts.

Space, topological: Nonmetrical space, for which certain axioms hold. Part-whole relation and connectedness are among its basic concepts.

Sum, topological: The sum of regions A and B ($A + B$) is the totality of the points which are included in either A or B.

Topological image: A one-to-one correspondent continuous image.

Topologically equivalent (isomorphous): A region is topologically equivalent to another if it is possible to convert the first into the second by a process of continuous transformation without changing the connection within the region, *i.e.*, by stretching or bending without tearing.

B. Psychological Concepts

1. Concepts Mainly Concerned with Directly Observable Facts

Action, representation of: Under some circumstances action is represented as a region; under others it is represented as a path.

Adit: A region perceived as permitting locomotion to a certain region.

Arm: A part of a region which, without separating from the region, makes a locomotion to or communicates with another region.

Behavior: By behavior we mean any change in the life space, which is subject to psychological laws (*see* Influences "from outside"). Behavior (B) at a given time is a function of the life space (L) at that time. $B = f(L)$.

Boundary, of a psychological region: Those points of a region for which there is no surrounding that lies entirely within the region. The presence of a boundary within the environment or person can be determined by means of locomotions or communications. A boundary of a psychological region is not necessarily an obstacle to locomotion or communication.

Boundary, sharp: Psychologically one can distinguish sharp and unsharp boundaries. In the case of a sharp boundary it can be determined for every point of the life space whether or not it belongs to the region in question.

Boundary zone (between region *M* and *N*): That region (*BZ*) which is foreign to *M* and *N* and which has to be crossed by a locomotion from *M* to *N*.

x **Differentiation:** The degree of differentiation refers to the number of subparts within a region. Under certain circumstances the degree of dynamical unity can be used as inverse criterion for the degree of differentiation.

Distance: Although "distance" is not a topological concept, distances in the life space can be compared by topological means if the path corresponding to one distance is a part of the path corresponding to the other distance.

Environment: Everything in which, toward which, or away from which the person can perform locomotions is part of the environment.

Field: Space, conceived as having a certain characteristic at every point.

x **Integration:** The process by which the number of subparts of a region is reduced.

Life space: Totality of facts which determine the behavior (*B*) of an individual at a certain moment. The life space (*L*) represents the totality of possible events. The life space includes the person (*P*) and the environment (*E*). $B = f(L) = f(P,E)$. It can be represented by a finitely structured space.

Life space, foreign hull of: Facts which are not subject to psychological laws but which influence the state of the life space.

Locomotion: Change of position. Locomotion can be regarded as a change of structure: the moving region becomes a part of another region. Locomotion can be represented by a path which can or cannot be carried out. This path characterizes a change of position within a field which otherwise remains sufficiently constant. One can distinguish quasi-physical, quasi-social, and quasi-conceptual locomotions.

Person: The person is represented as a differentiated region of the life space; however in the first approximation he can be represented as an undifferentiated region or a point.

Person, parts of: (1) Motor-perceptual stratum (region); (2) inner-personal stratum (region): (*a*) peripheral regions, (*b*) central regions. The motor-perceptual stratum has the position of a boundary zone between the inner-personal regions and the environment.

Point: From a psychological point of view the main property of a point is its character as an unstructured region.

Position, determination of: The position of a point in the life space is characterized by the region which includes it. The exactness of the determination depends upon the extent to which one can distinguish subregions within the region in question.

Region, connected psychological: *A* and *B* are parts of a connected psychological region (1) if a locomotion from *A* to *B* is possible without leaving the region; (2) if a change of state of *A* leads to a change of state of *B*.

Region, determination of: (1) A psychological region can be determined by its qualitative properties and by the topological relations of the region or of its boundary to other regions or their boundaries; (2) by psychological processes which connect different points, especially by locomotions or communications.

Region, psychological: Part of the life space. Everything that is represented as a region in characterizing a psychological situation must be a part of the life space. A region is not necessarily a connected one.

Region of undetermined quality: A region whose cognitive structure or quality is not sufficiently determined for the individual. In some respects, it has dynamically the character of a barrier.

Regions, neighboring: Regions which have a common boundary (boundary zone).

Regions, overlapping: Regions which have a common part.

x **Restructuring:** Change of the relative position of part regions without change of their number.

Situation: Life space or part of it conceived in terms of its content (meaning). The life space may consist of one situation or of two or more overlapping situations. The term situation refers either to the general life situation or the momentary situation.

Situation, overlapping: Two or more situations which exist simultaneously and which have a common part. The person is generally located within this common part.

Space of free movement: Regions accessible to the person from his present position. The space of free movement is usually a multiply connected region. Its limits are determined mainly by (1) what is forbidden to a person, (2) what is beyond his abilities.

Structure of a region: Refers to (1) degree of differentiation of the region, (2) arrangement of its part regions, (3) degree of connection between its part regions.

2. Dynamic Concepts

Accessibility: The ease with which a region can be reached by locomotion or communication. Degree of accessibility can be represented by barriers or walls. The accessibility of a person can be represented by boundaries of different strengths between the environment and certain inner regions of the person.

Barrier: A boundary (boundary zone) which offers resistance to locomotion. The degree of this resistance can be different (1) for different kinds of locomotion, (2) for locomotion in different directions, (3) at different points of the barrier.

Barrier, impassable: A boundary (boundary zone) which is impassable for the locomotion in question.

Barrier, inhomogeneous: A barrier which offers different resistance at different points.

Barrier, outer: A barrier encircling the person.

Communication: Two regions are in communication if a change of the state of one region changes the state of the other. The degree of communication corresponds to the degree of dynamical dependence. The degree of communication depends upon (1) the kind of communicating processes, (2) the properties of the communicating regions, and (3) the boundary (boundary zone) between them. The degree of communication from A to B need not be the same as that from B to A. Communication can be represented by a boundary zone (wall); a low degree of communication corresponds to a strong wall.

Elasticity: The tendency of a changed region to return to its original state. The degree of elasticity can differ for regions of the same fluidity.

Equilibrium: A constellation of forces such that the forces at a point are opposite in direction and equal in strength.

Fluidity: The smaller the forces necessary (other conditions being equal) to produce a certain change, the more fluid a thing or medium is. The fluidity of the same region can be different for different kinds of influences.

Force: Cause of change; a basic concept of vector psychology. Properties of a force are: strength, direction, and point of application. Strength and direction can be represented by a vector.

Friction: The resistance which a passable region shows to locomotion.

Gestalt: A system whose parts are dynamically connected in such a way that a change of one part results in a change of all other parts. This unity may differ for different kinds of changes.

Irreality: *see* Reality

Material properties: Those factors determining the state of a region which are conceived as properties of the region itself.

Medium: A region in or through which locomotions can be carried out.

Need: A need corresponds to a tension system of the inner-personal region.

Plasticity: The plasticity of a region corresponds to the ease of producing a relatively lasting and stable change in its structure.

Power field: The sphere of influence of a person. It can be represented as a field of inducing forces.

Reality, degree of: A property of psychological facts. Differences in degree of reality can be coordinated to a special dimension of the life space. The more irreal levels show a greater fluidity. The structure of a more real level depends less upon the will of the person. The degree to which the life space is structured in the dimension reality-irreality depends upon the specific character, *e.g.*, age, of the person and the momentary situation.

Regression: Corresponds to a decrease in the differentiation of the person.

Rigidity: Boundaries (barriers, walls) are the more rigid the greater the forces necessary to overcome them. Rigidity of a region can differ for different types of processes.

Solidity: *see* Rigidity.

Structure, cognitive: Structure of the life space corresponding to the knowledge of the person.

Substitute value: An action *b* has a substitute value for the action *a* if the tension of the system corresponding to *a* is released when the tension of the system corresponding to *b* is released.

System: A region considered in regard to its state, especially to its state of tension.

Tension: A state of a region relative to surrounding regions. It involves forces at the boundary of the region which tends to produce changes such that differences of tension are diminished.

Thing: A region in or through which locomotions cannot be carried out.

Valence: A valence corresponds to a field of forces whose structure is that of a central field. One can distinguish positive and negative valences.

Wall: Boundary (boundary zone) considered as to its influence on communication. A stronger wall corresponds to a smaller degree of communication.

Weight, relative: Relative importance of one of two or more overlapping situations (regions) within a life space.

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